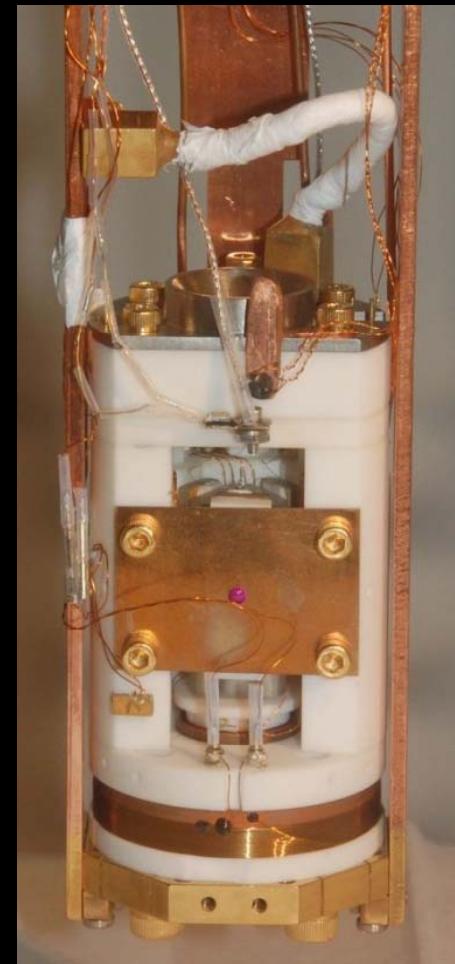
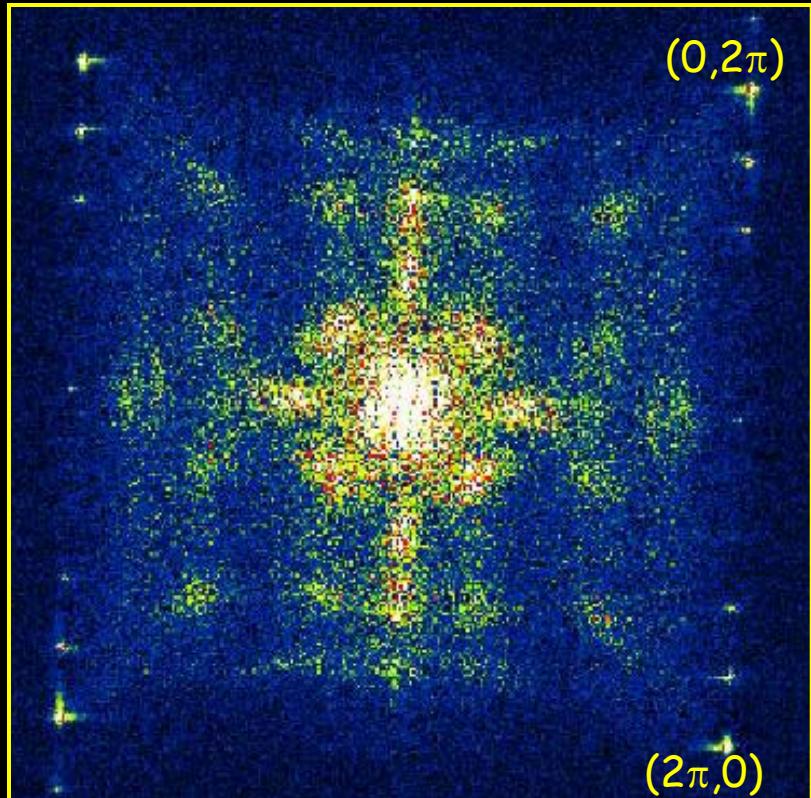


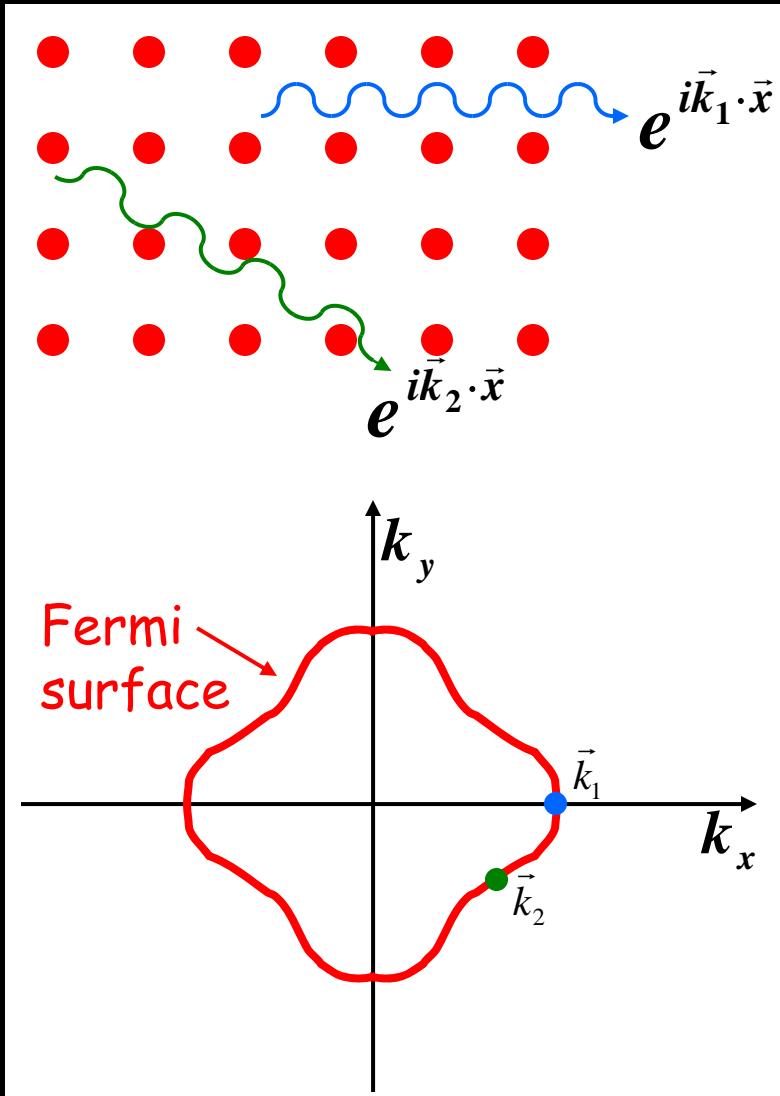
# STM as a Probe of Real and Momentum Spaces in Correlated Electron Systems



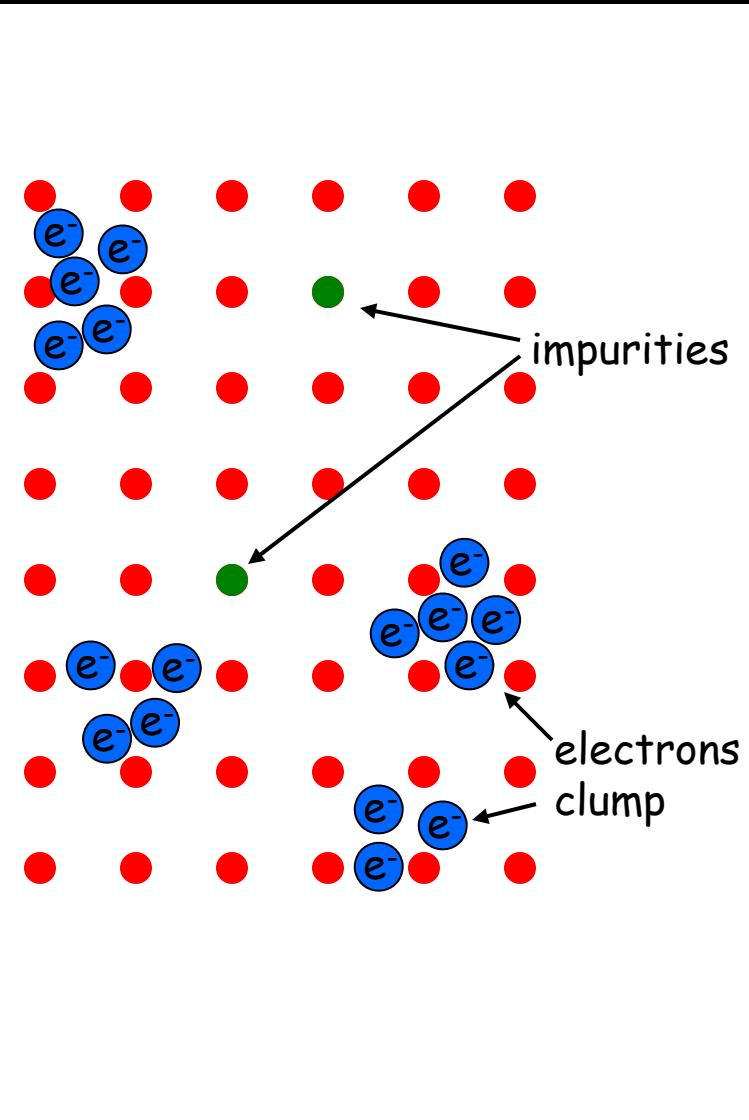
## Outline

- Pedagogical explanation of Quasiparticle Interference (QPI)
- Review of the last 8 years of QPI imaging in cuprate superconductors
  - Controversies in the cuprates:
    - Static vs. dispersing order
    - Pseudogap: competing phase vs. superconducting fluctuations
- Conclusions: we've made progress, but still not sure
  - Need to track QPI and checkerboards through  $T^*$
  - Need to understand spatial phases of patterns
  - Need to fully understand the artifacts of STM setup conditions
- QPI in magnetic field
- QPI on iron-pnictide superconductors

Simple clean metals:  
all info is in momentum space



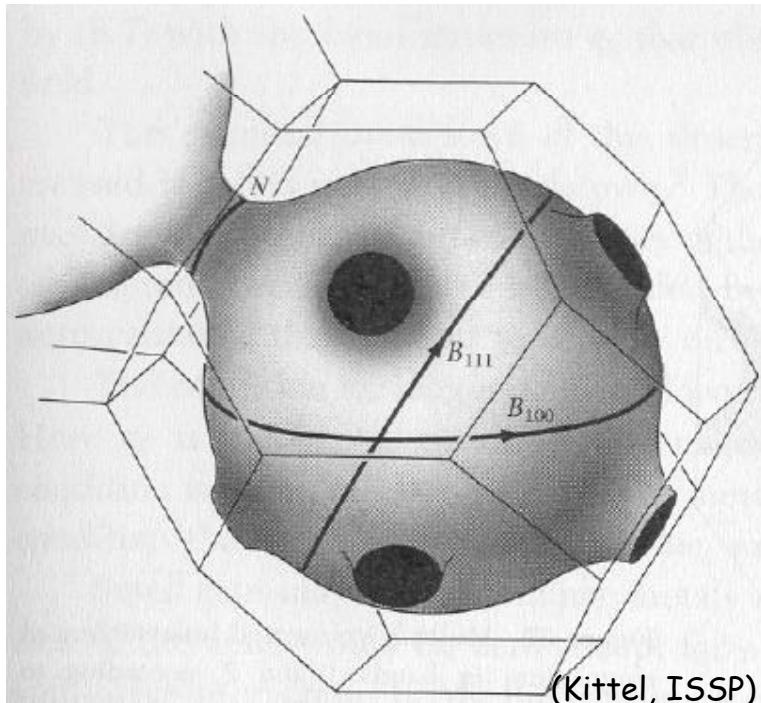
More complex materials:  
info is in real space inhomogeneity



Simple clean metals:  
all info is in momentum space

With no impurities, there is  
nothing to see in real space

Fermi surface of Cu:



Bloch wave functions:

$$\Psi_k(\vec{r}) = u_k(\vec{r}) \cdot e^{i\vec{k} \cdot \vec{r}}$$

$\vec{k}$  = electron momentum

For a given energy  $E$ , add up  
all the states in  $k$ -space:

$$DOS(E, \vec{r})$$

$$\propto \sum_k |\Psi_k(\vec{r})|^2 \delta(E - \varepsilon(\vec{k}))$$

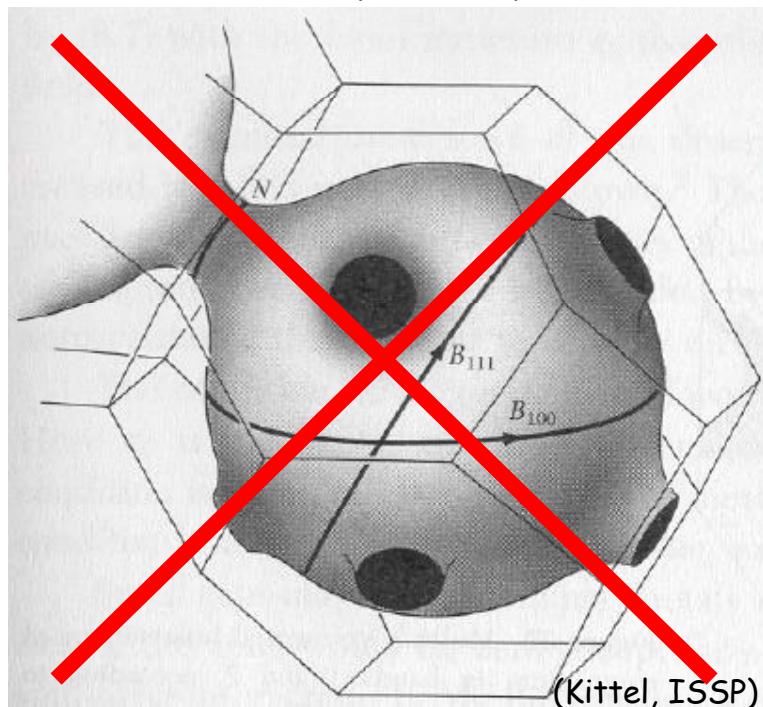
Bloch wave function:

$$\Psi_k(\vec{r}) = u_k(\vec{r}) \cdot e^{i\vec{k} \cdot \vec{r}}$$

DOS( $r$ ) is uniform in a metal !

**BORING!**

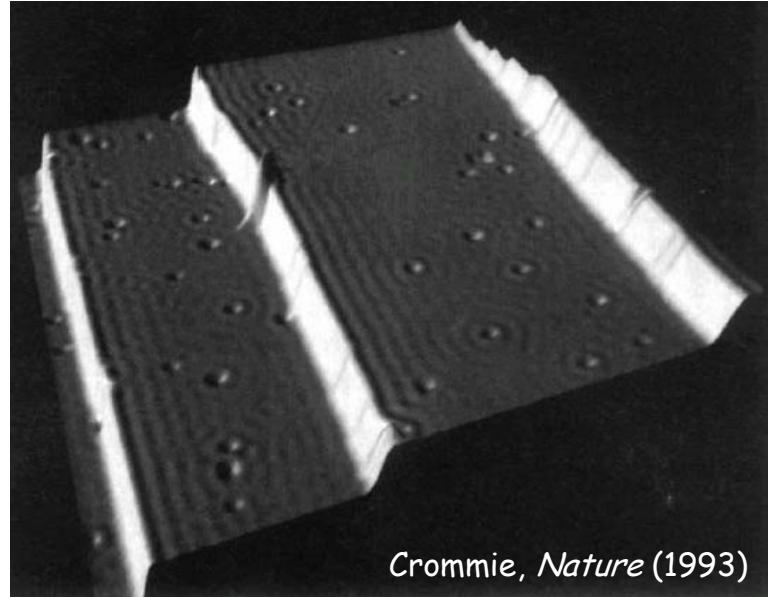
Dirty metal:  
potential no longer periodic



$\vec{k}$  is a wavevector, so the system must uniform over several  $\lambda = 2\pi/k$  in order for  $k$  to have any meaning

Image impurities in real space

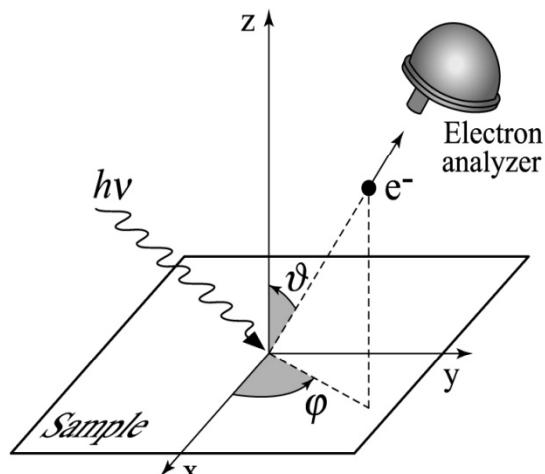
Real surface of Cu:



Crommie, *Nature* (1993)

with step edges  
and impurities

## Momentum space tool: Angle-Resolved Photoemission Spectroscopy (ARPES)



Photoemission geometry

Conserve energy:

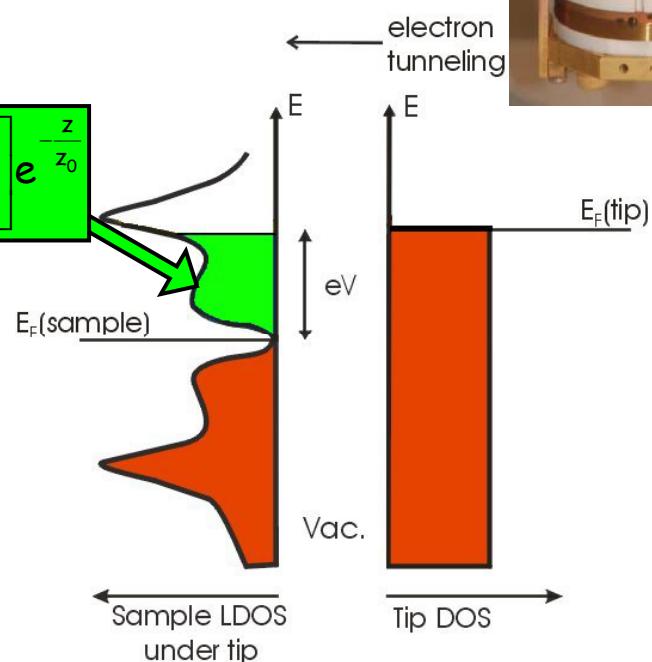
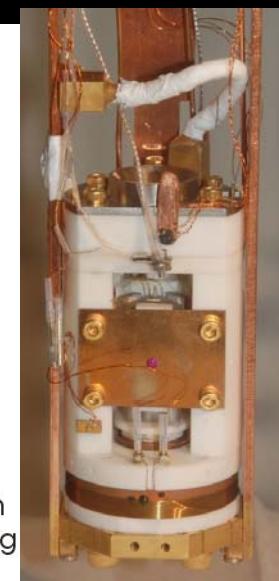
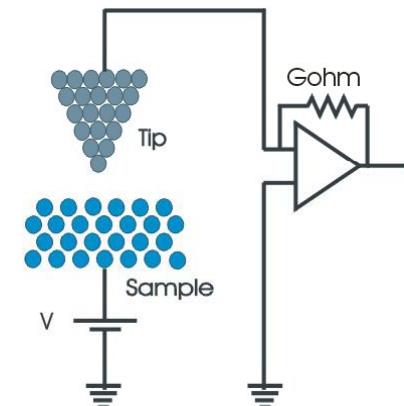
$$E_{kin} = h\nu - \phi - |E_B|$$

Conserve momentum:

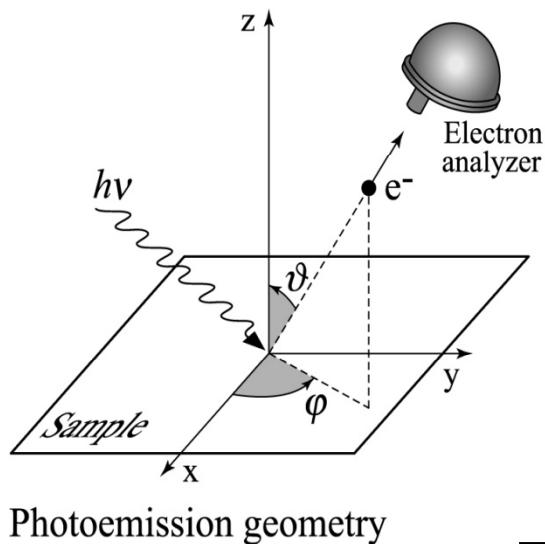
$$\vec{p}_{\parallel} = \hbar \vec{k}_{\parallel} = \sqrt{2m_e E_{kin}} \sin \vartheta$$

No spatial resolution:  
averages over  $> (100 \text{ } \mu\text{m})^2$

## Real space tool: Scanning Tunneling Microscopy /Spectroscopy (STM/STS)



## Momentum space tool: Angle-Resolved Photoemission Spectroscopy (ARPES)



Conserve energy:

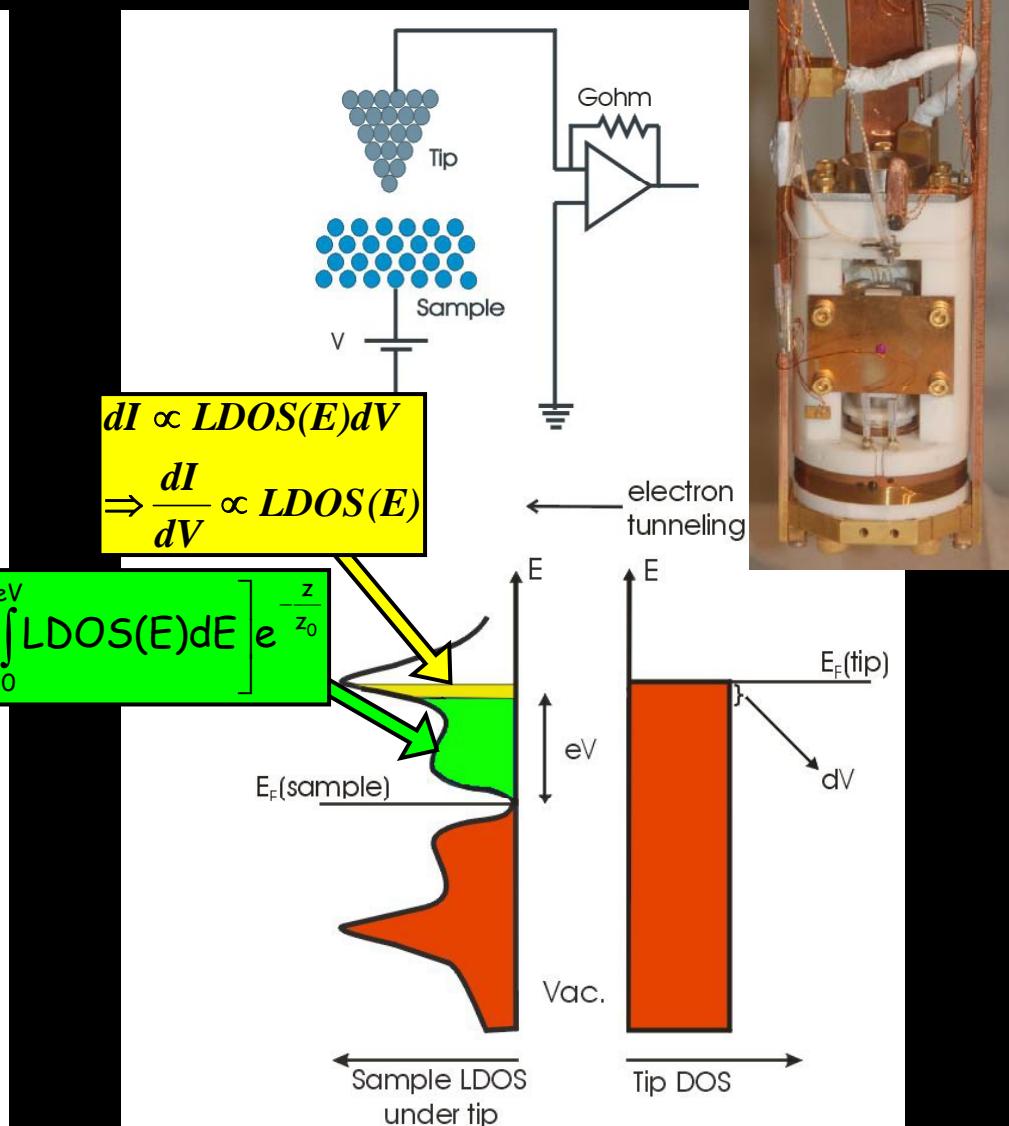
$$E_{kin} = h\nu - \phi - |E_B|$$

Conserve momentum:

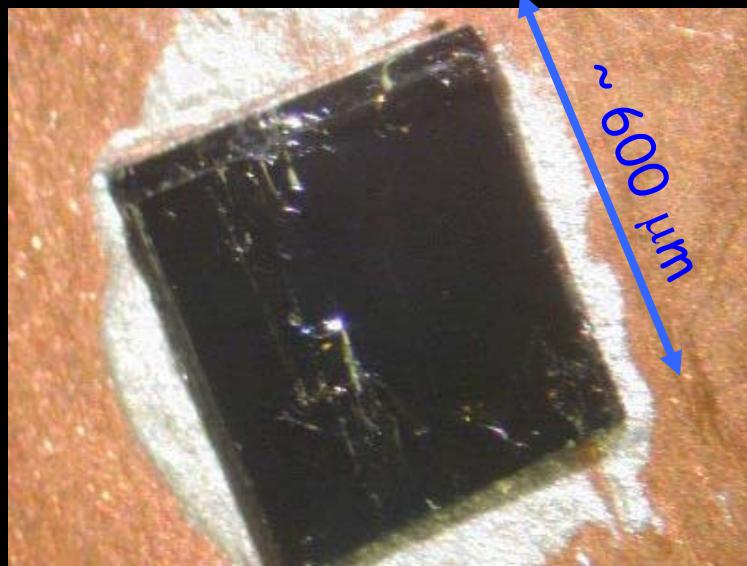
$$\vec{p}_{\parallel} = \hbar \vec{k}_{\parallel} = \sqrt{2m_e E_{kin}} \sin \vartheta$$

No spatial resolution:  
averages over  $> (100 \text{ }\mu\text{m})^2$

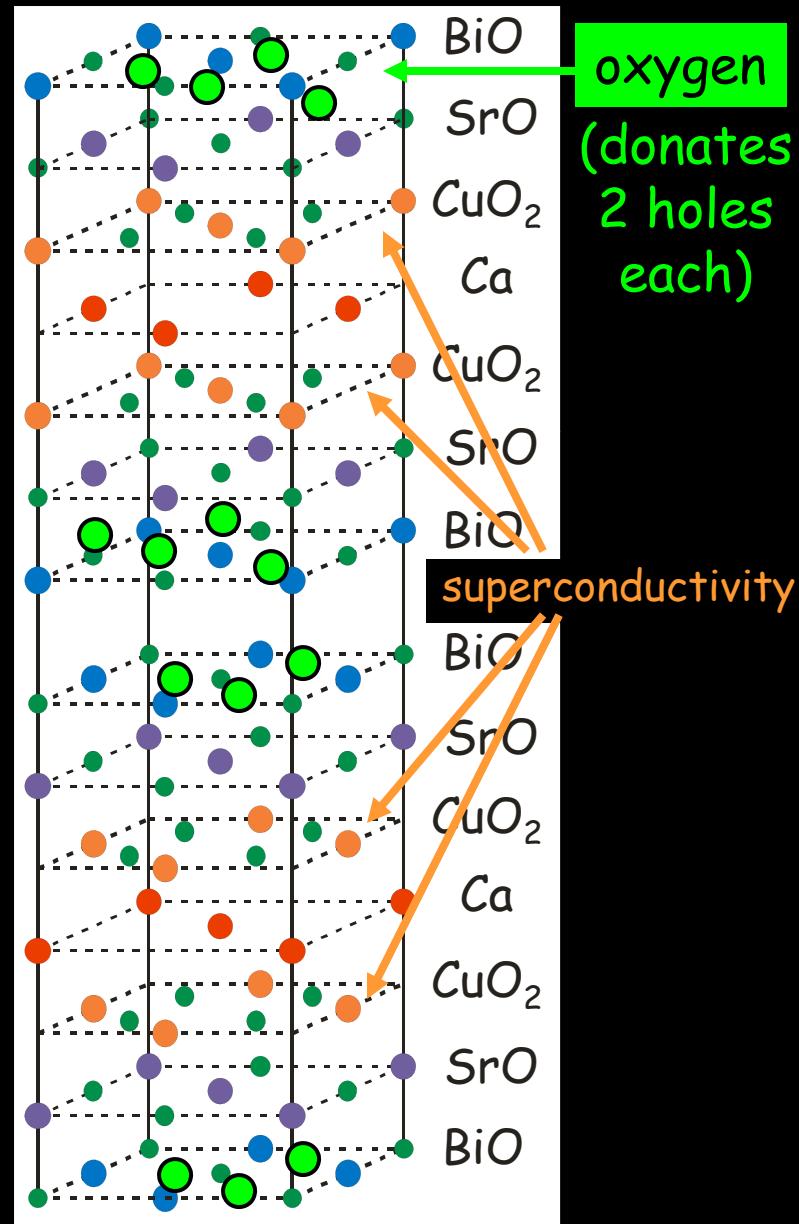
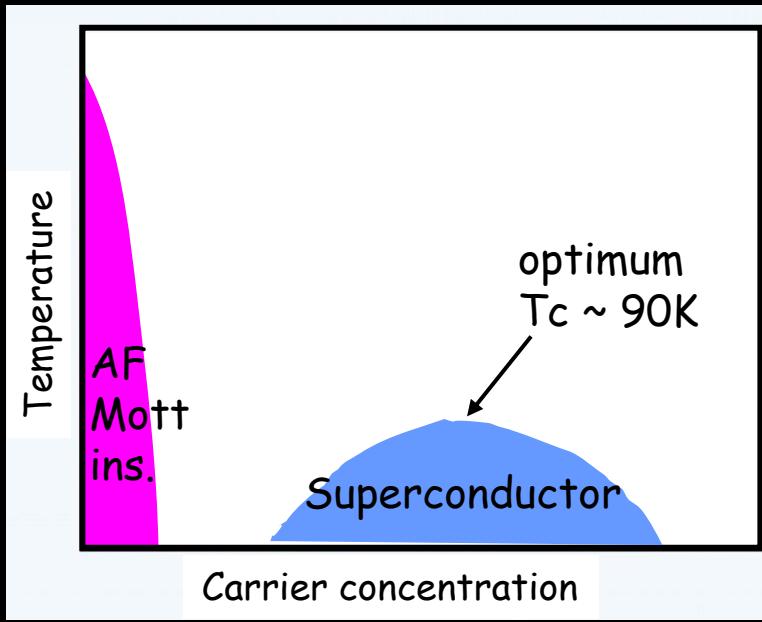
## Real space tool: Scanning Tunneling Microscopy /Spectroscopy (STM/STS)



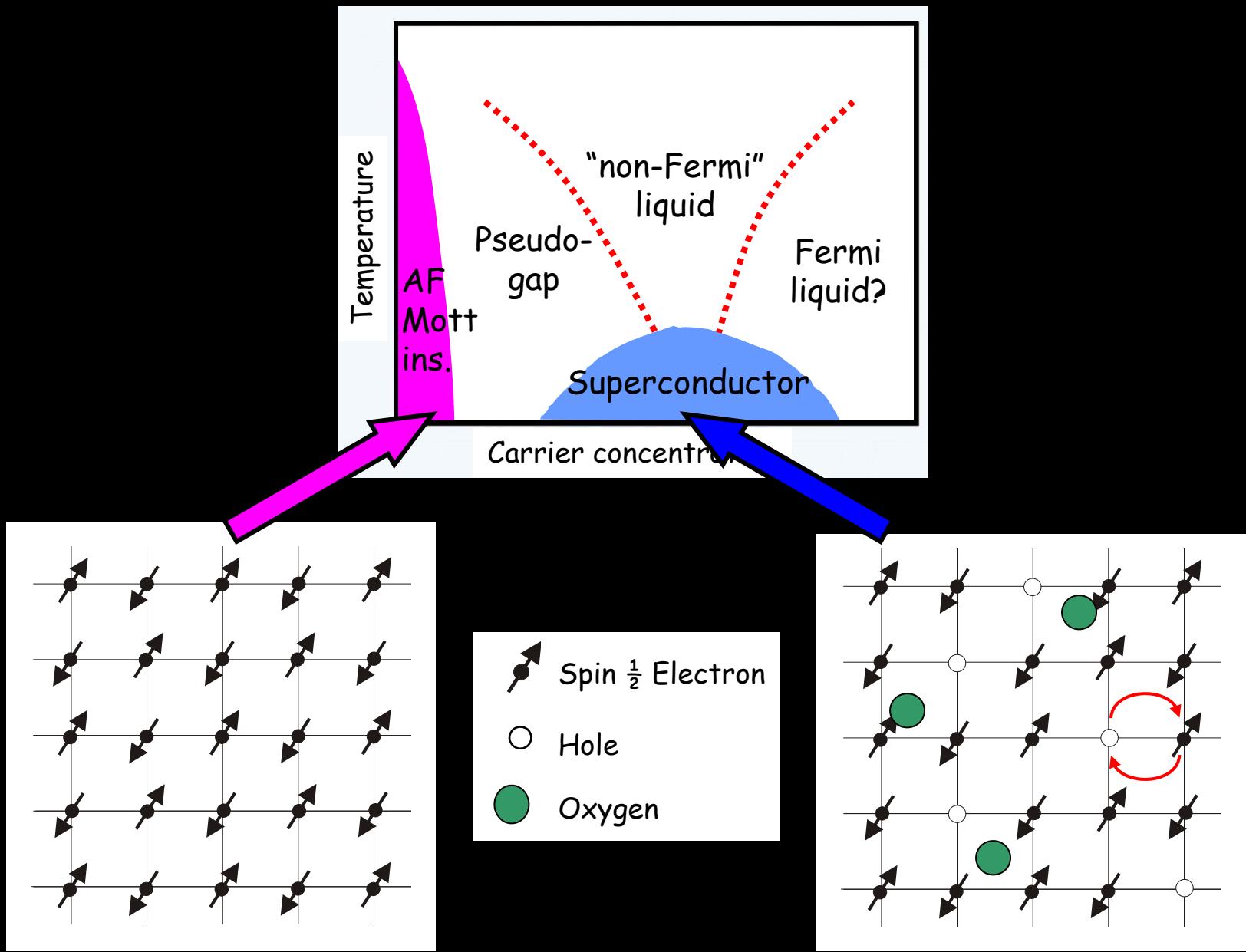
# $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ : a High-Tc Superconductor



High Tc cuprate phase diagram:

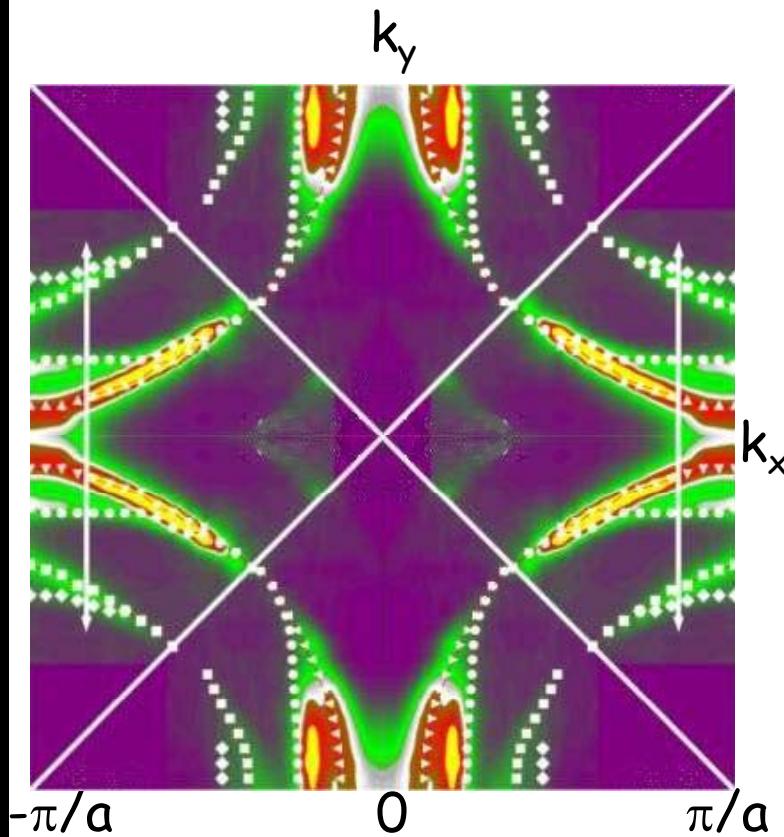


# High Temperature Superconductor: Doped Mott Insulator



Momentum space tool:  
Angle-Resolved Photoemission  
Spectroscopy (ARPES)

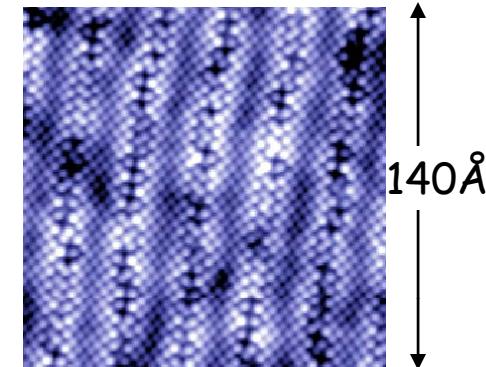
1st Brillouin zone:



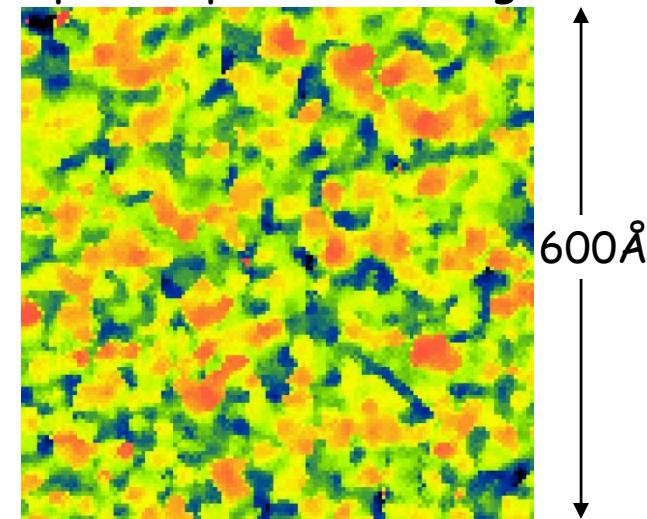
D.L. Feng, PRL (2001)

Real space tool:  
Scanning Tunneling Microscopy  
/Spectroscopy (STM/STS)

topography:



map of superconducting  $\Delta$ :



# Summary of experimental techniques:

## Momentum space:

- ARPES (Angle-Resolved Photo Emission Spectroscopy)

## Real space:

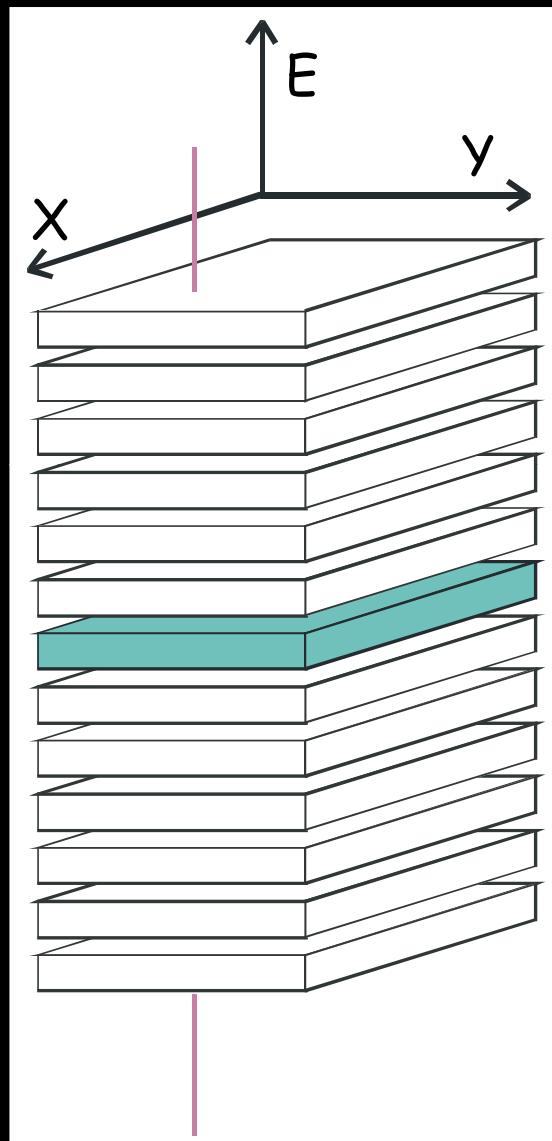
- STM (Scanning Tunneling Microscopy)

## R-space & k-space:

- FT-STS (Fourier Transform Scanning Tunneling Spectroscopy)

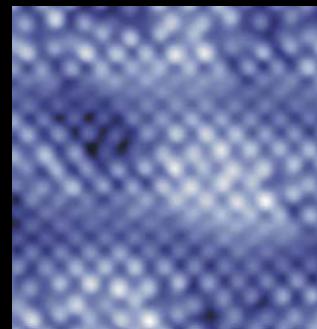
# Traditional STM/STS Measurements

Local Density of States ( $X, Y, E$ )



Constant current mode:

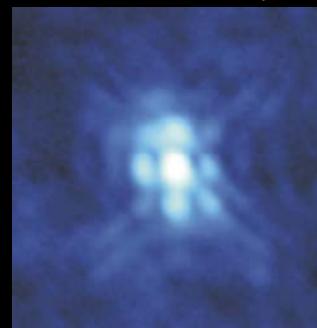
Topography



2 Å  
0 Å

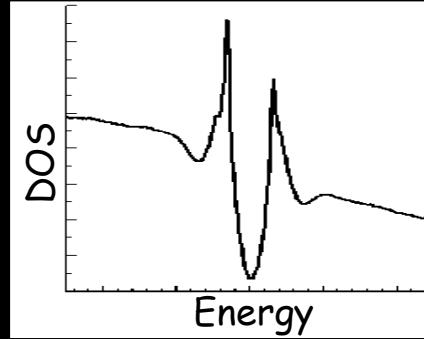
STM

$dI/dV$  Map



STM  
/STS

$dI/dV$  Spectrum

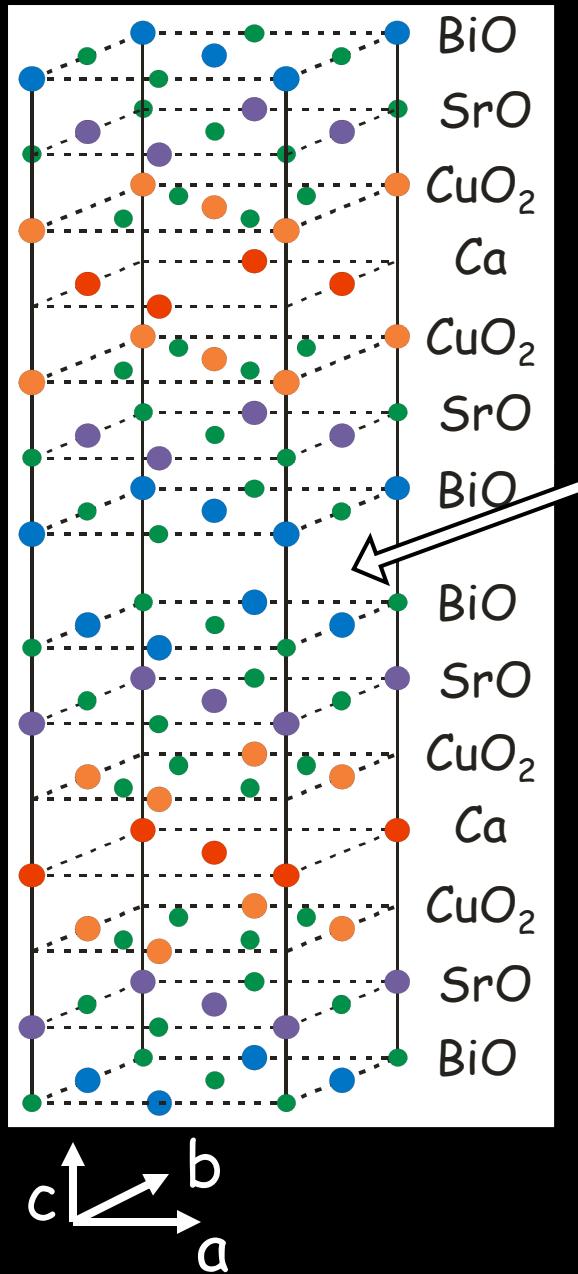


STS

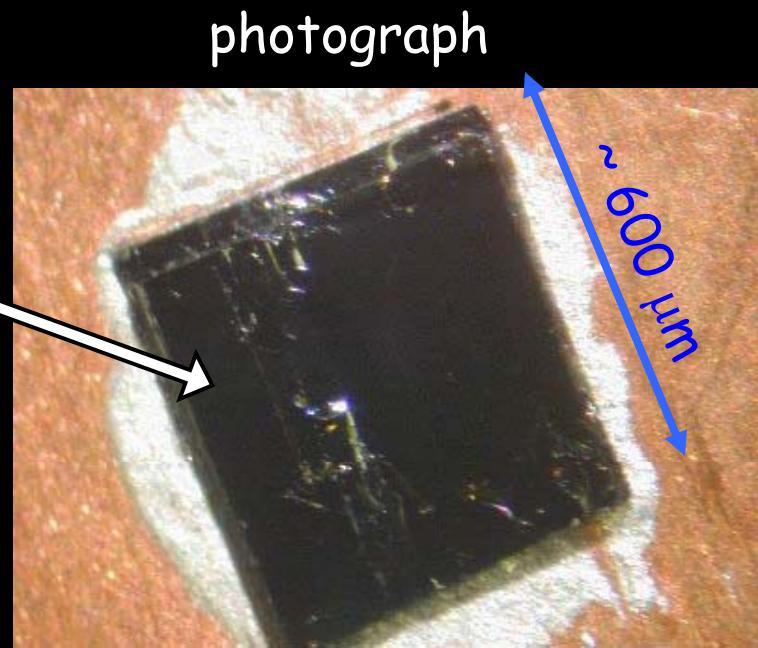
Scanning Tunneling Microscopy:

R-space

# Structure of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$

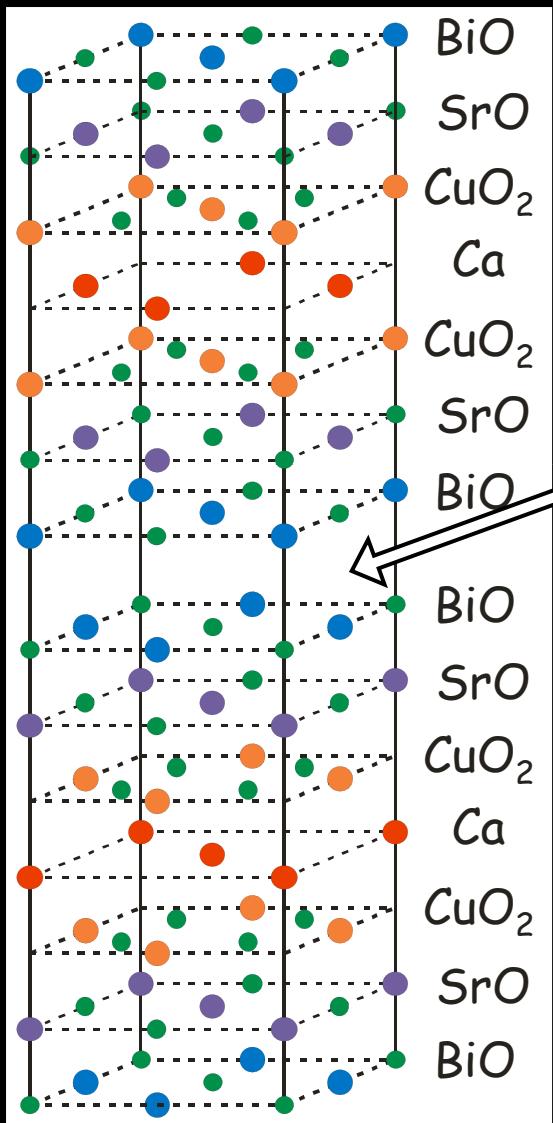


Cleave Here  
Reveals  
 $\text{BiO}$  Surface

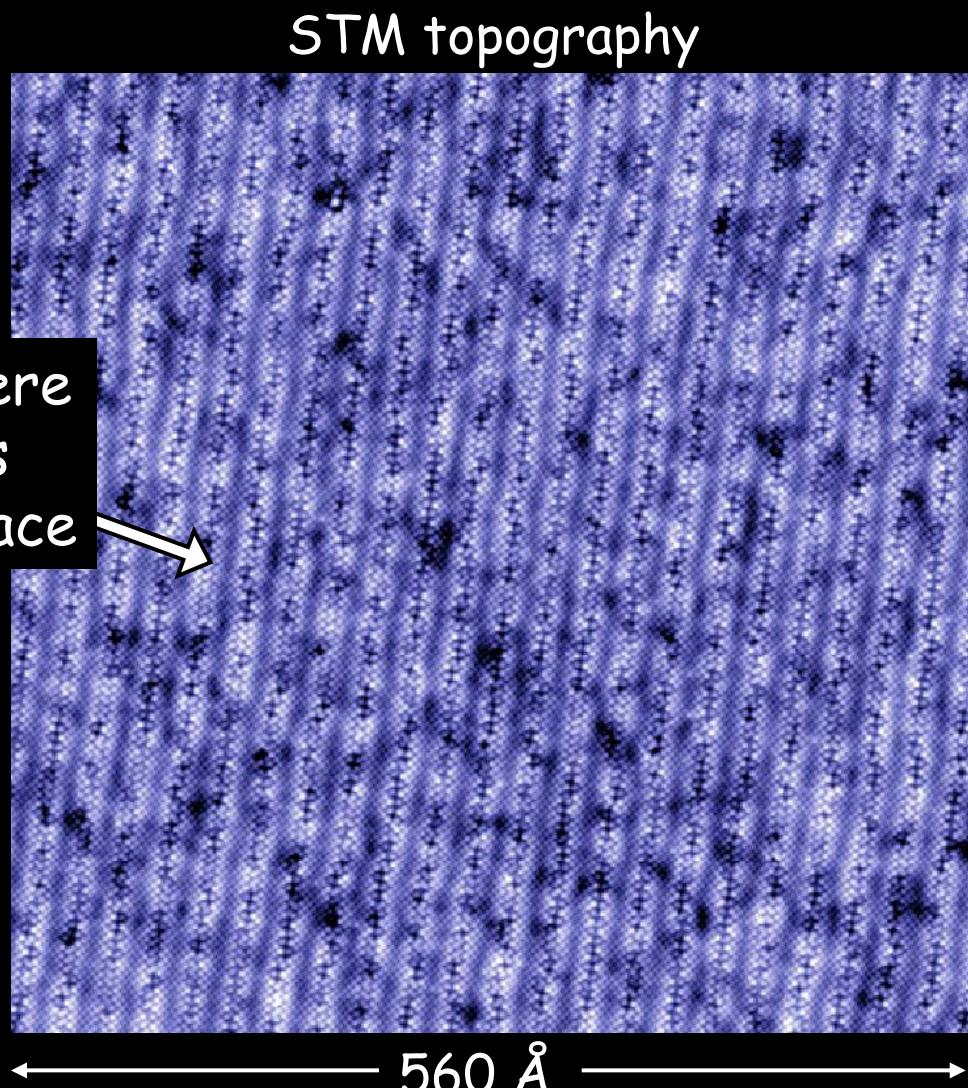


$T_c \sim 90 \text{ K}$

# Atomic resolution $\text{Bi}_2\text{Sr}_2\text{Ca}\text{Cu}_2\text{O}_{8+\delta}$



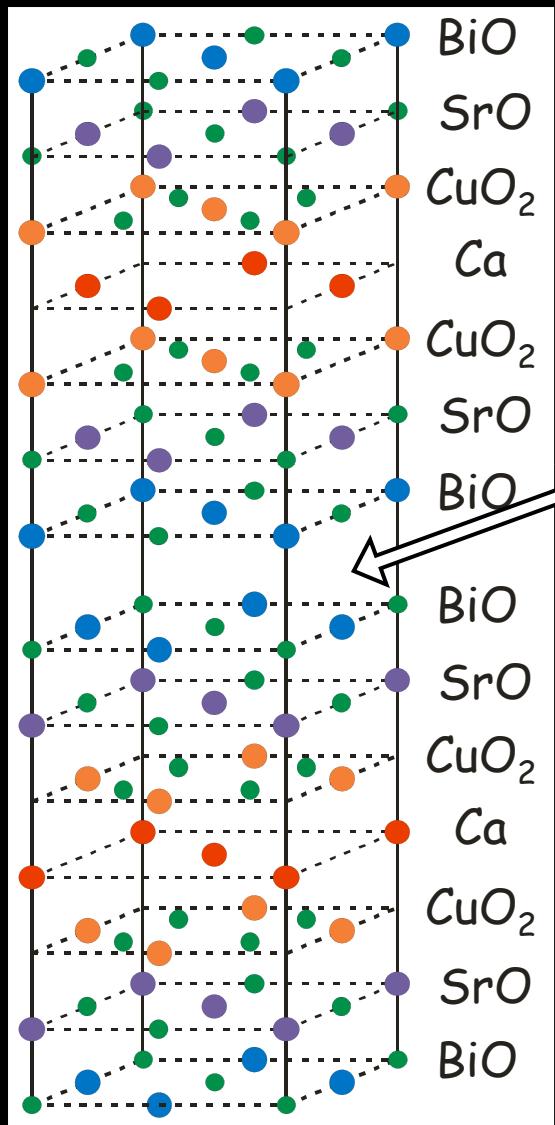
Cleave Here  
Reveals  
 $\text{BiO}$  Surface



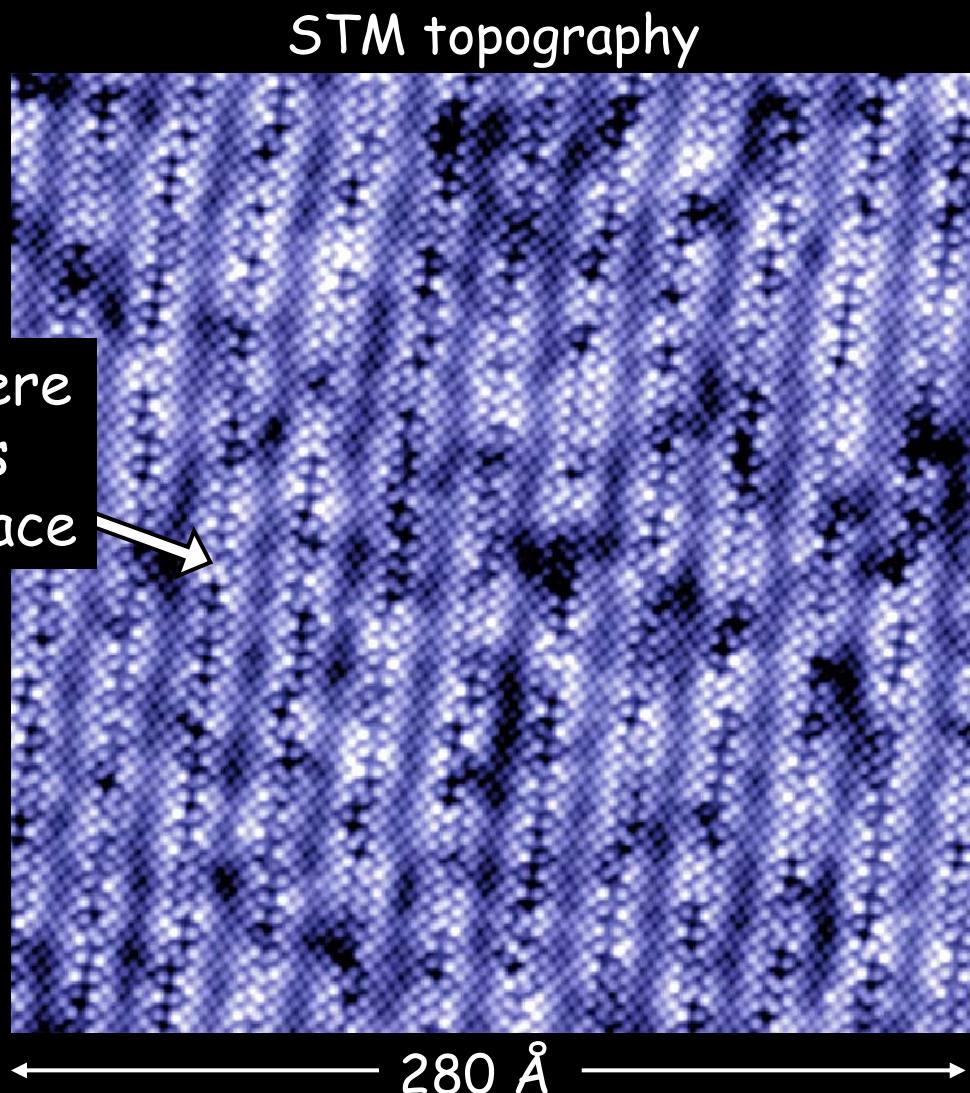
$c$   
 $b$   
 $a$

$T = 4.2\text{ K}, B = 0\text{ T}$   
 $100\text{ pA}, -100\text{ mV}$

# Atomic resolution $\text{Bi}_2\text{Sr}_2\text{Ca}\text{Cu}_2\text{O}_{8+\delta}$



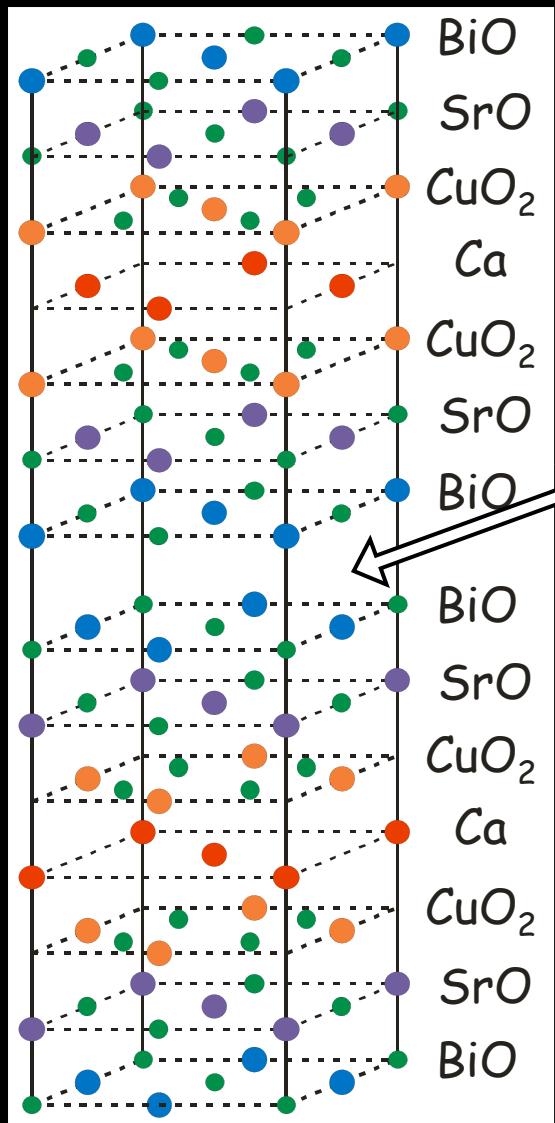
Cleave Here  
Reveals  
 $\text{BiO}$  Surface



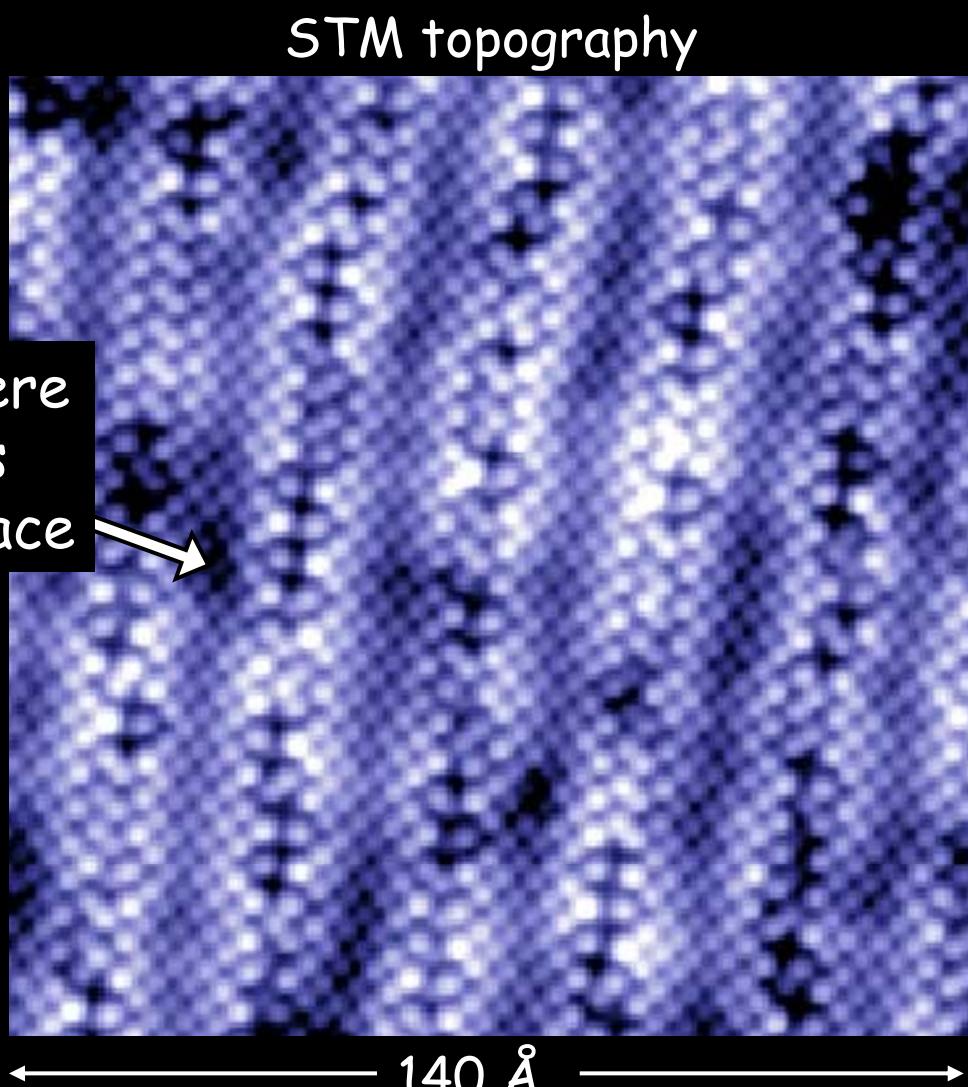
$c$   
 $b$   
 $a$

$T = 4.2\text{ K}, B = 0\text{ T}$   
 $100\text{ pA}, -100\text{ mV}$

# Atomic resolution $\text{Bi}_2\text{Sr}_2\text{Ca}\text{Cu}_2\text{O}_{8+\delta}$



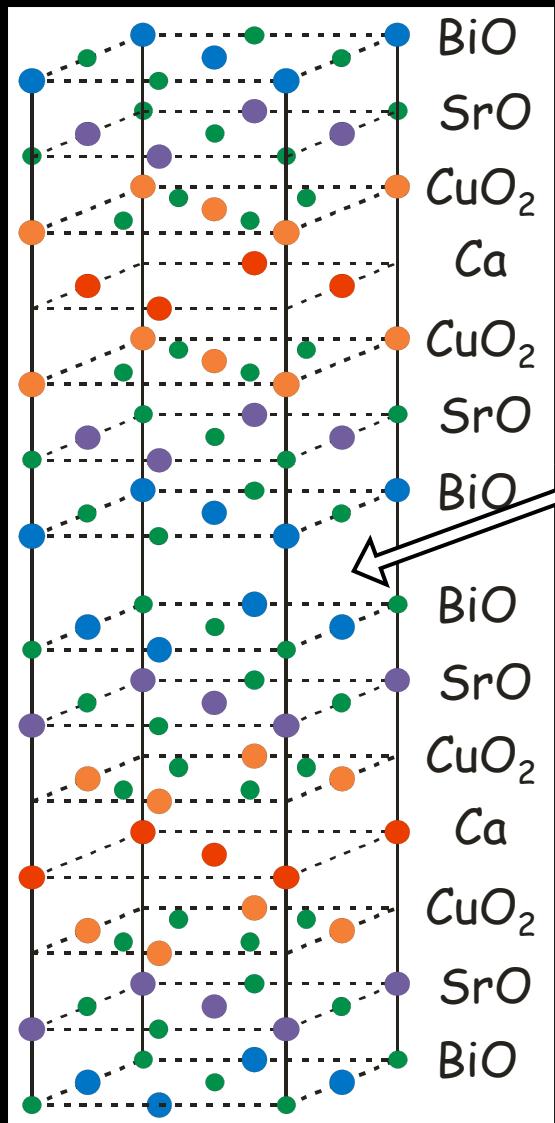
Cleave Here  
Reveals  
 $\text{BiO}$  Surface



$c$   
 $b$   
 $a$

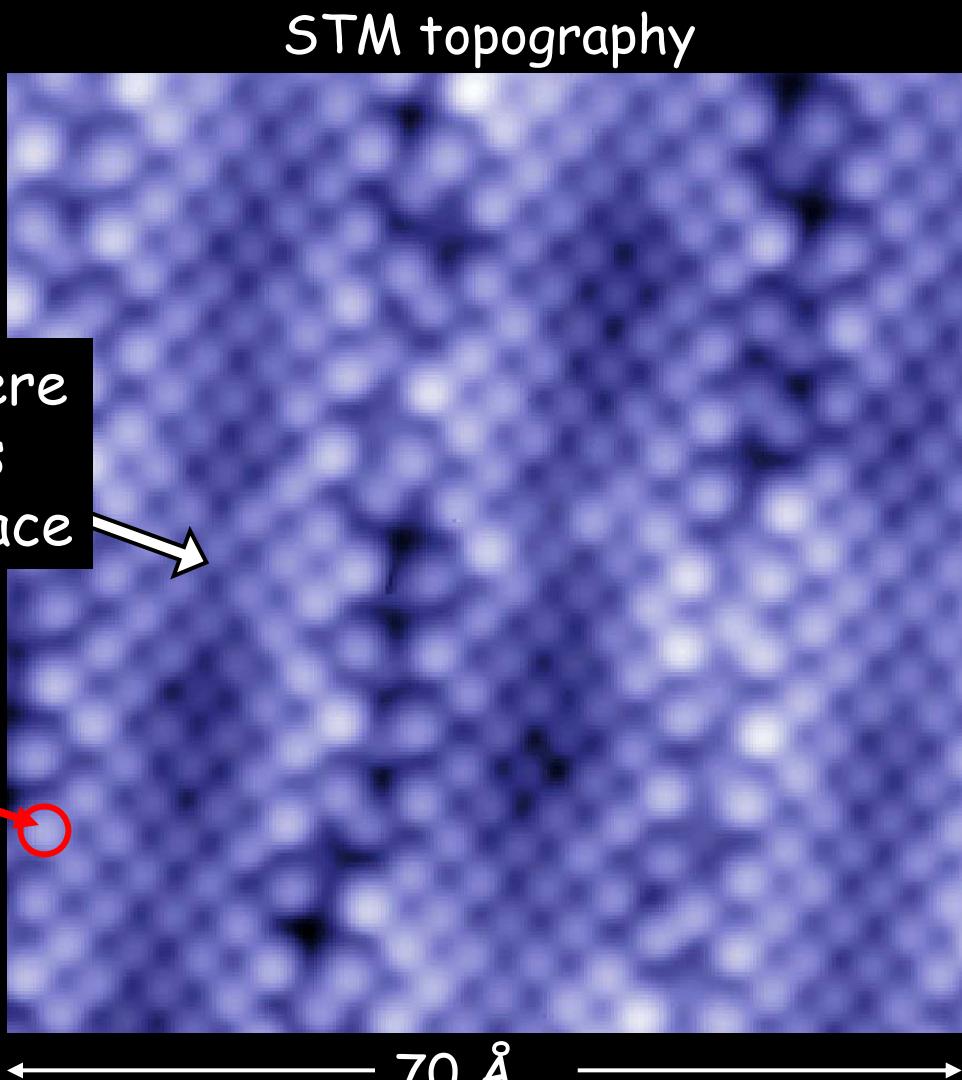
$T = 4.2\text{ K}, B = 0\text{ T}$   
 $100\text{ pA}, -100\text{ mV}$

# Atomic resolution $\text{Bi}_2\text{Sr}_2\text{Ca}\text{Cu}_2\text{O}_{8+\delta}$



Cleave Here  
Reveals  
BiO Surface

single  
Bi atom



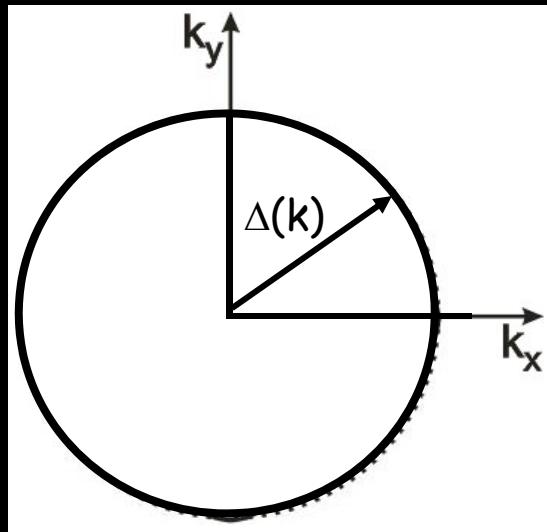
$c$   
 $b$   
 $a$

$T = 4.2\text{K}, B = 0\text{T}$   
 $100\text{pA}, -100\text{mV}$

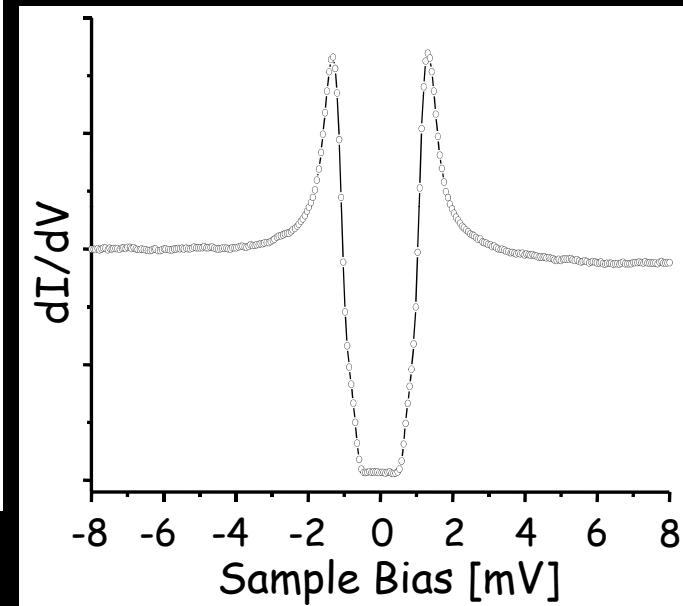
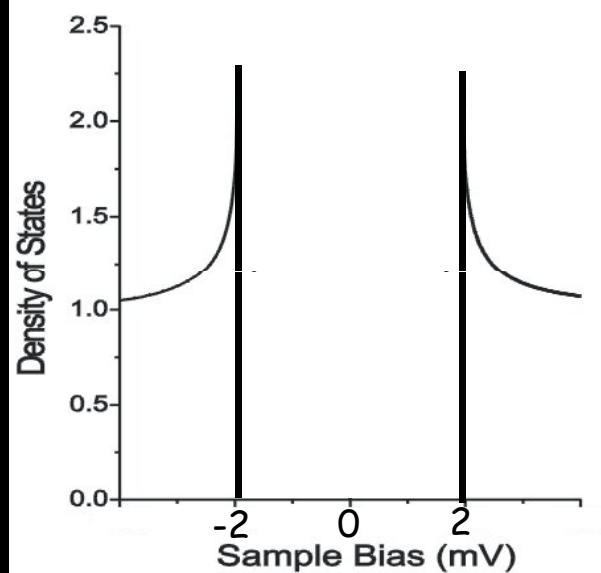
# Scanning Tunneling Spectroscopy: energy dependence

# Expected DOS for a conventional s-wave superconductor

Gap Magnitude vs.  $k$



DOS( $E$ ) after average over  $k$

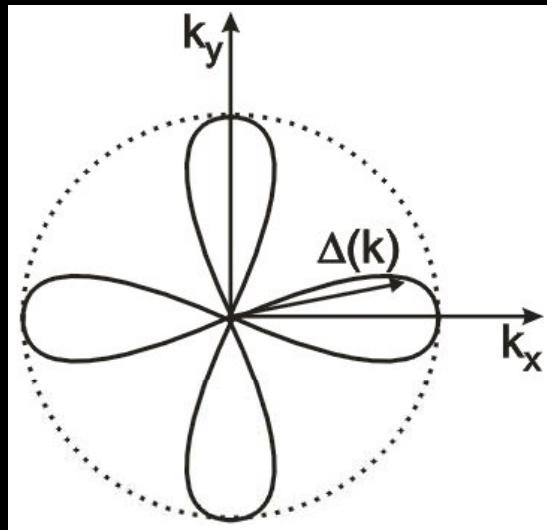


STM Experiment  
measured on  $NbSe_2$

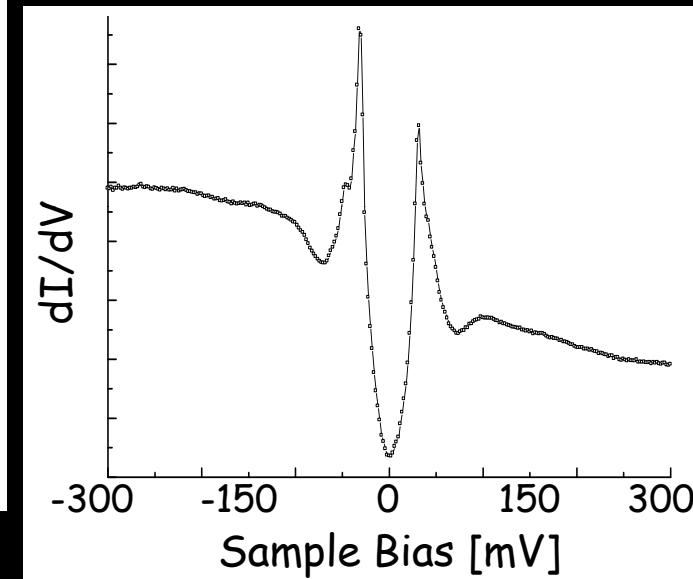
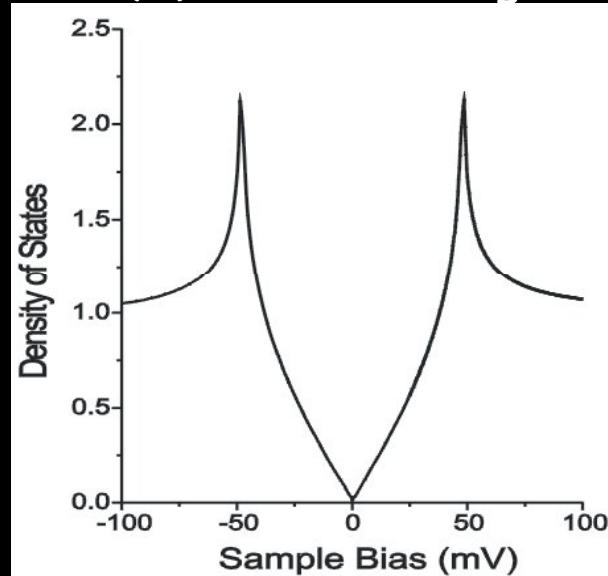
Rev. Sci. Inst. 70, 1459 (1999).

# Expected k-averaged DOS for a $d_x^2 - d_y^2$ Superconductor

Gap Magnitude vs.  $k$



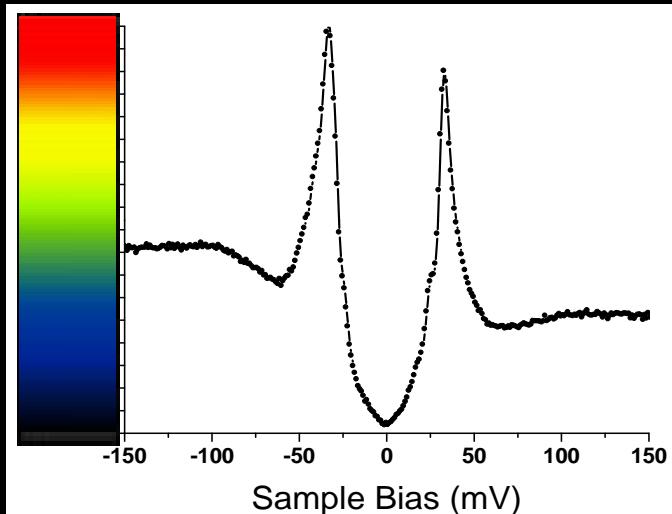
DOS( $E$ ) after average over  $k$



STM Experiment  
measured on BiO plane  
Typical Gap  $\approx 45$  meV  
[as-grown  $T_c \approx 89$ K]

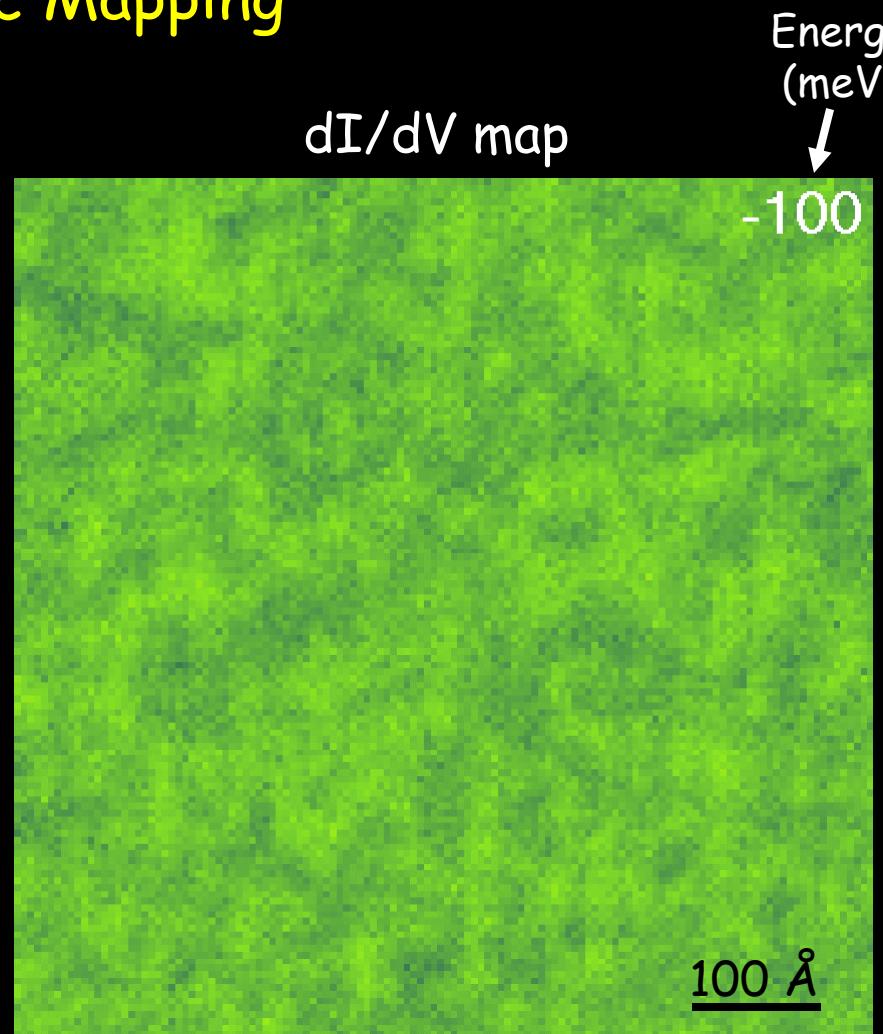
# Spectroscopic Mapping

Point dI/dV Spectrum



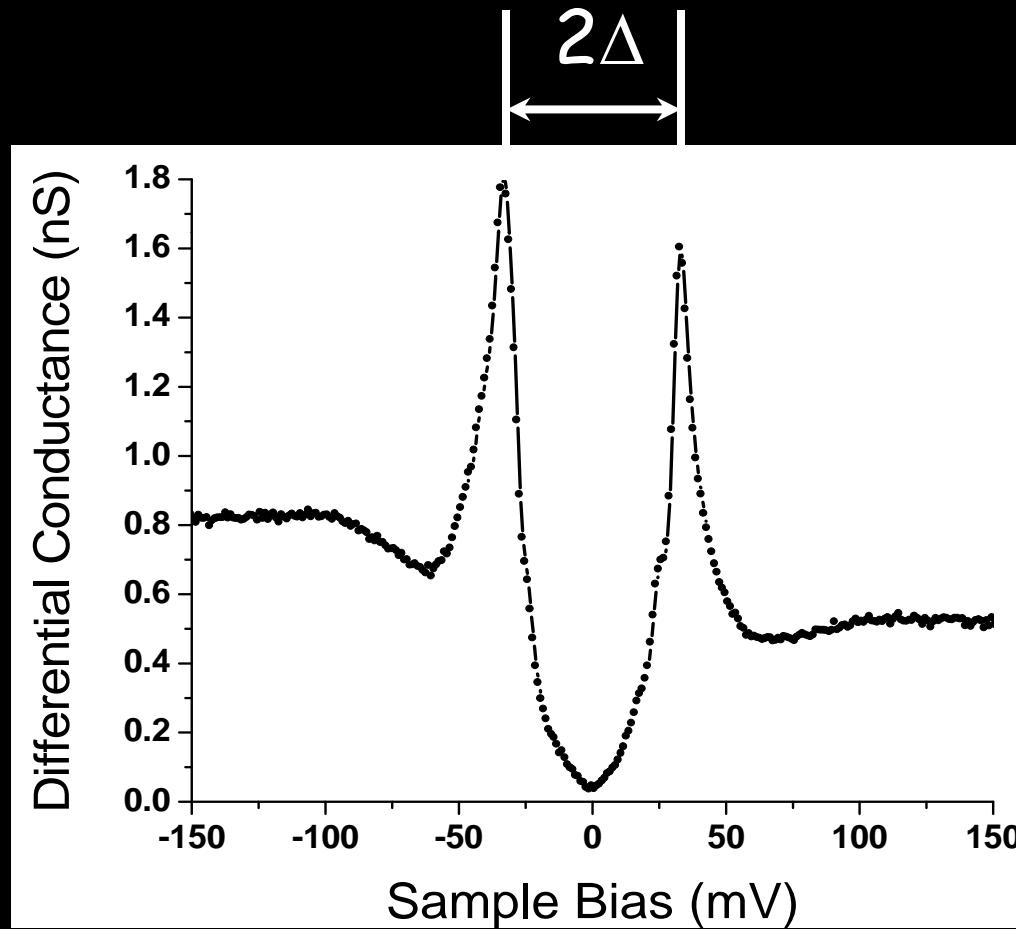
100 energies

at every  
location



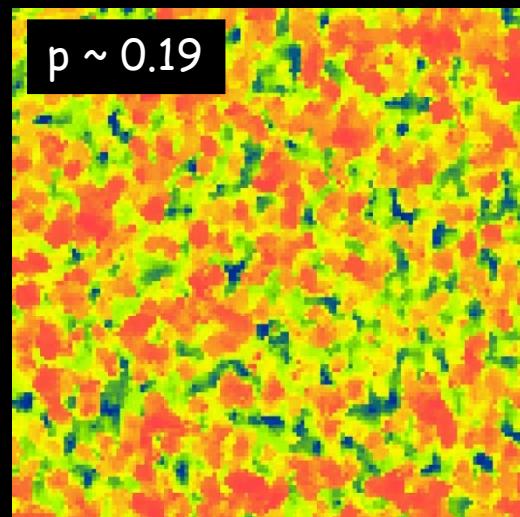
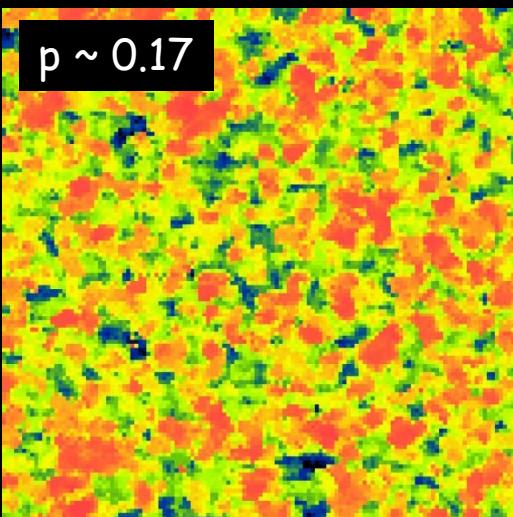
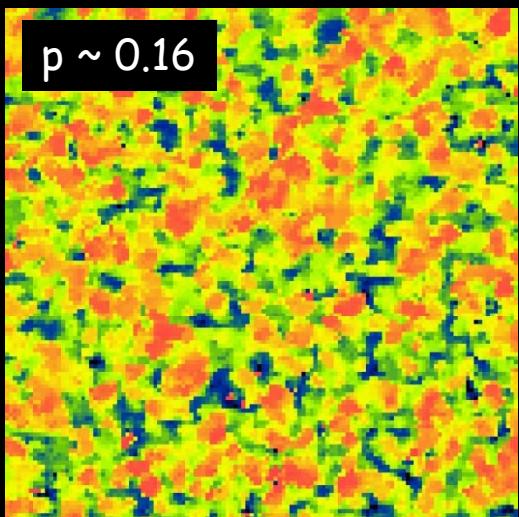
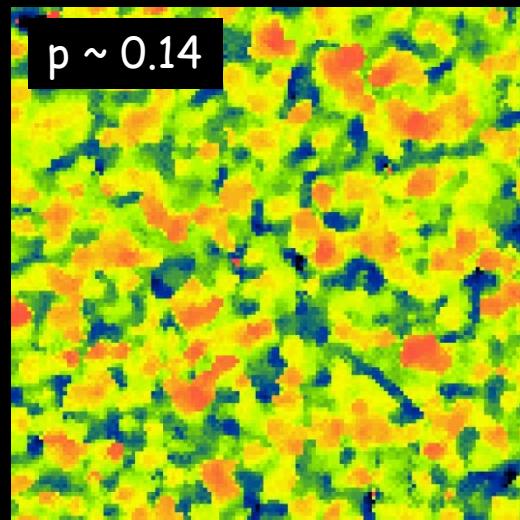
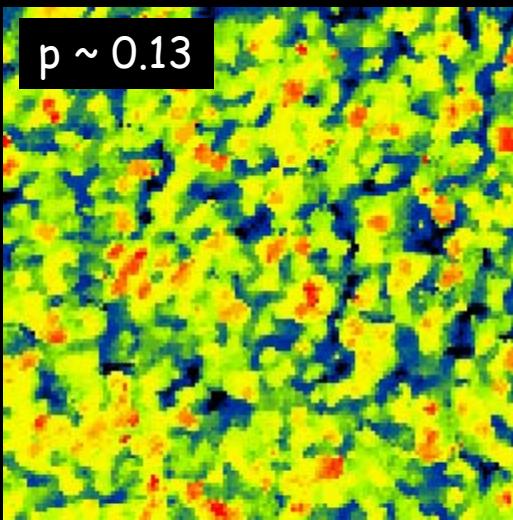
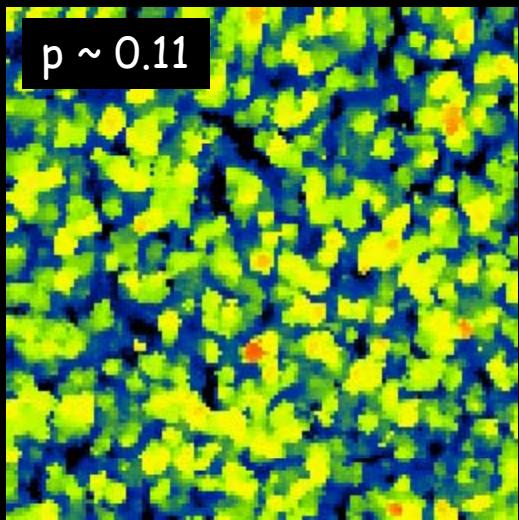
- $100 \times 128 \times 128 = 1,600,000$  measurements
- Registered to surface with atomic resolution for 2 days

# *GapMap:* Map of $\Delta$ as a function of location



# Evolution of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ gapmap with doping

70 mV  
20 mV



↔  
 $\sim 600 \text{ \AA}$

Nature 413, 282 (2001)

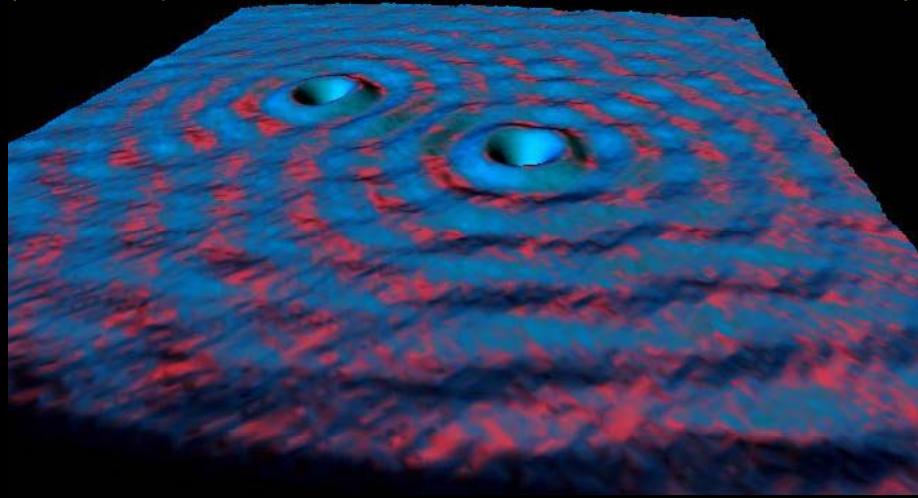
Nature 415, 412 (2002)

STM measurements so far: all in real space

So much disorder !

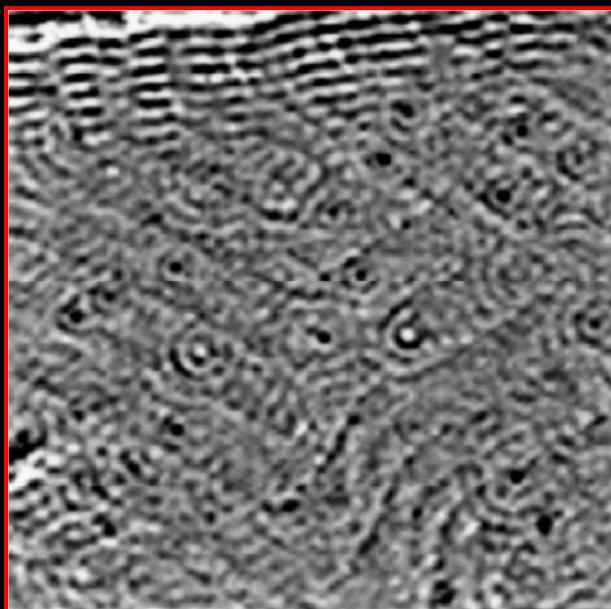
This disorder gives us the key to turn STM/STS  
into a simultaneous R-space and k-space probe

# Experiment: Quasiparticle Interference at Impurity Atoms in Metals (real space)

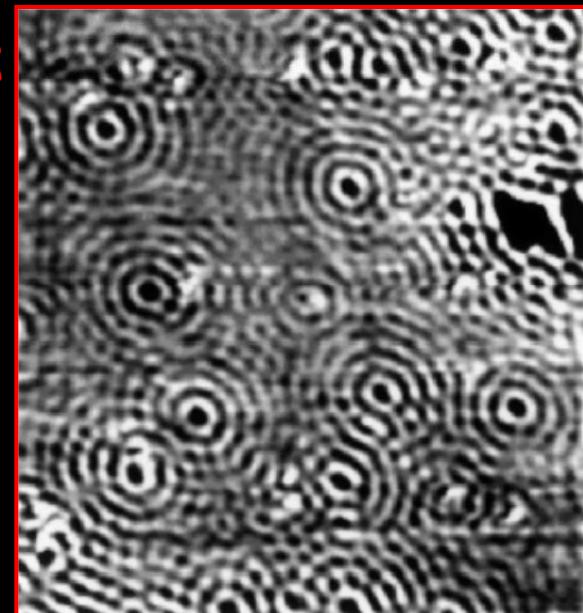


Crommie, Lutz & Eigler, Nature 363, 524 (1993)

Au:



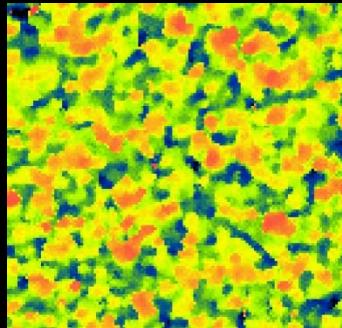
Cu:



Peterson, Hofmann, Plummer & Besenbacher, J. Electron Spectroscopy 109, 97 (2000)

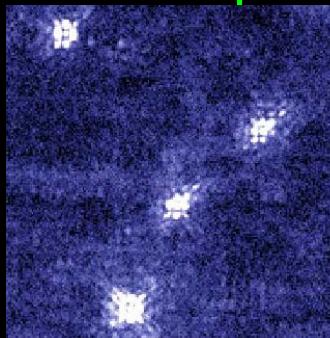
# Theory: Quasiparticle Interference from disorder scattering in the cuprates

Suppose you have some disorder potential  $V$ , such as



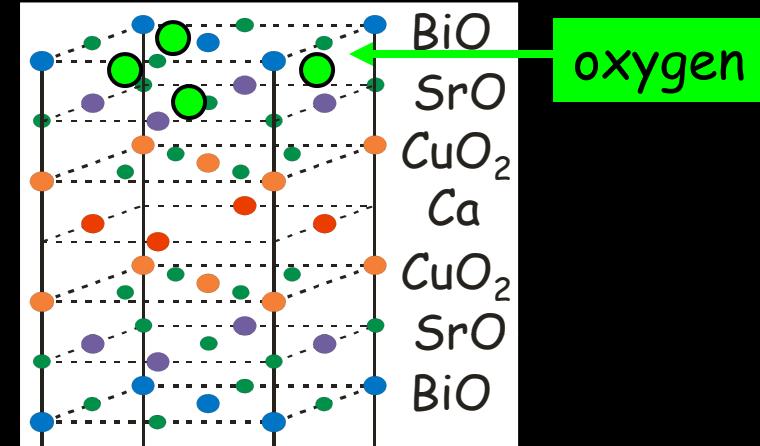
gap disorder

or



point defects

or



Each Fourier component  $V(\vec{q})$  will cause elastic scattering between initial and final states whose momenta differ by  $\vec{q}$ .

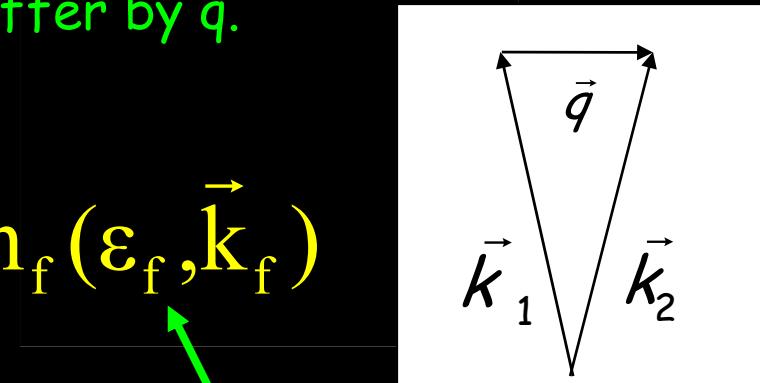
Power spectrum of scattering:

$$P(\varepsilon, \vec{q}) \propto |V(\vec{q})|^2 n_i(\varepsilon_i, \vec{k}_i) n_f(\varepsilon_f, \vec{k}_f)$$

Structure factor  
of disorder potential

Density of  
initial states

Density of  
final states

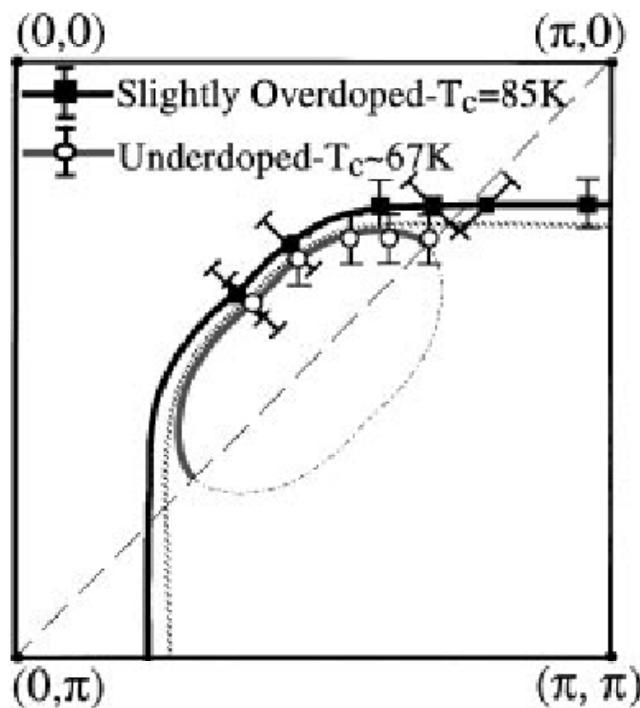


ARPES:

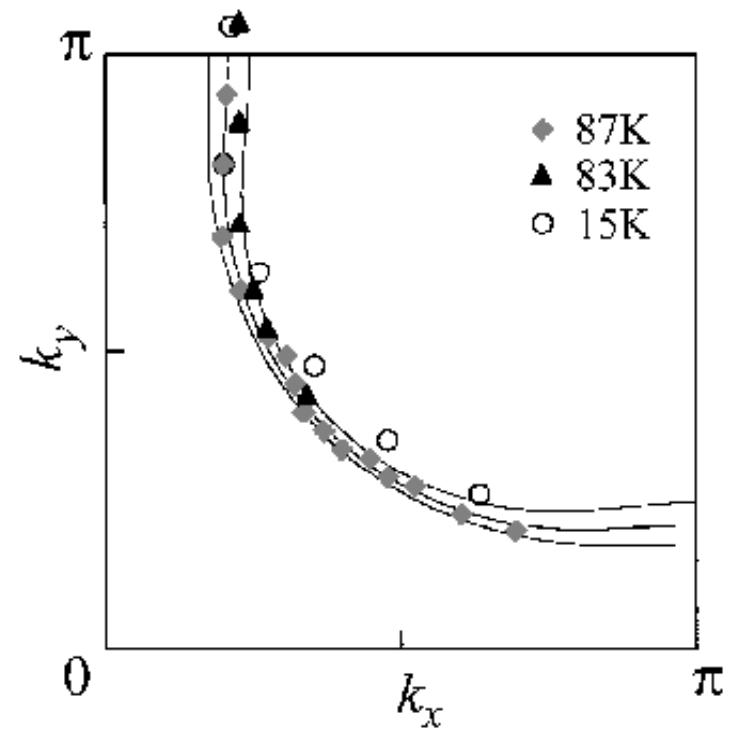
*k*-space

# ARPES: Normal State Fermi Surface

Marshall *et al.*, PRL 76, 4841 (1996)

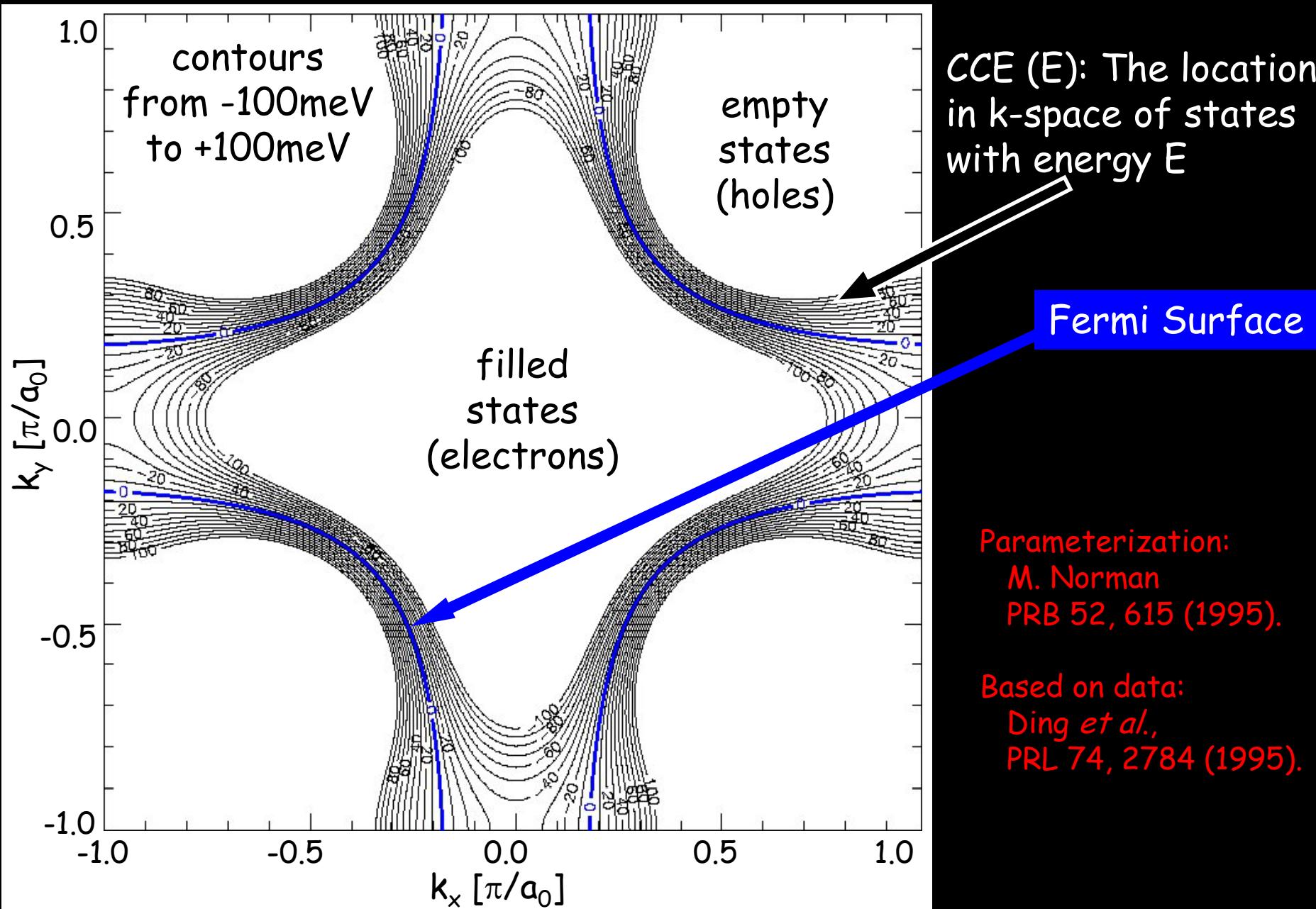


Ding *et al.*, PRL 78 2627 (1997)

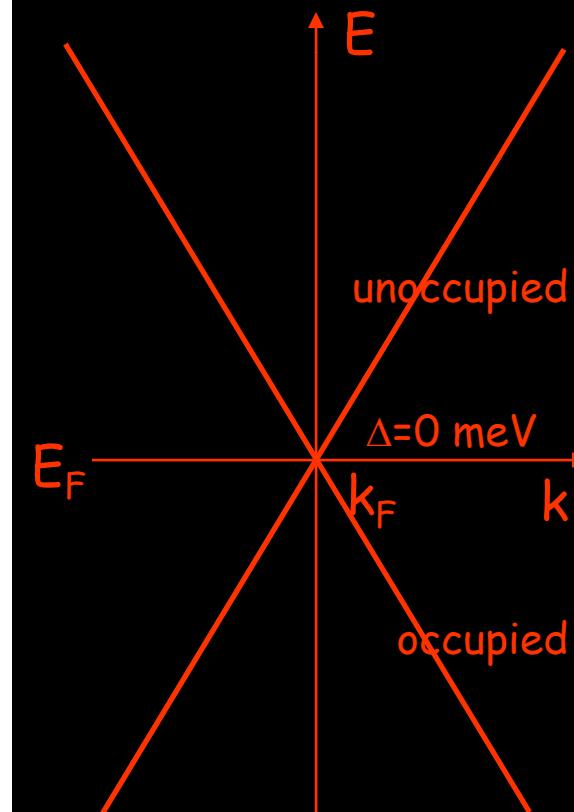
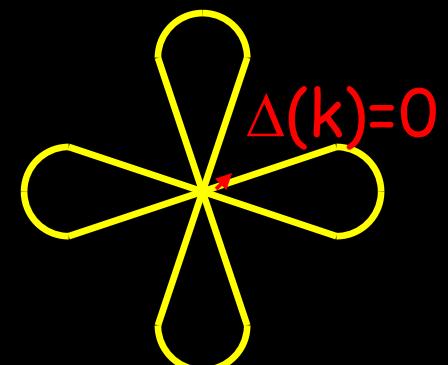
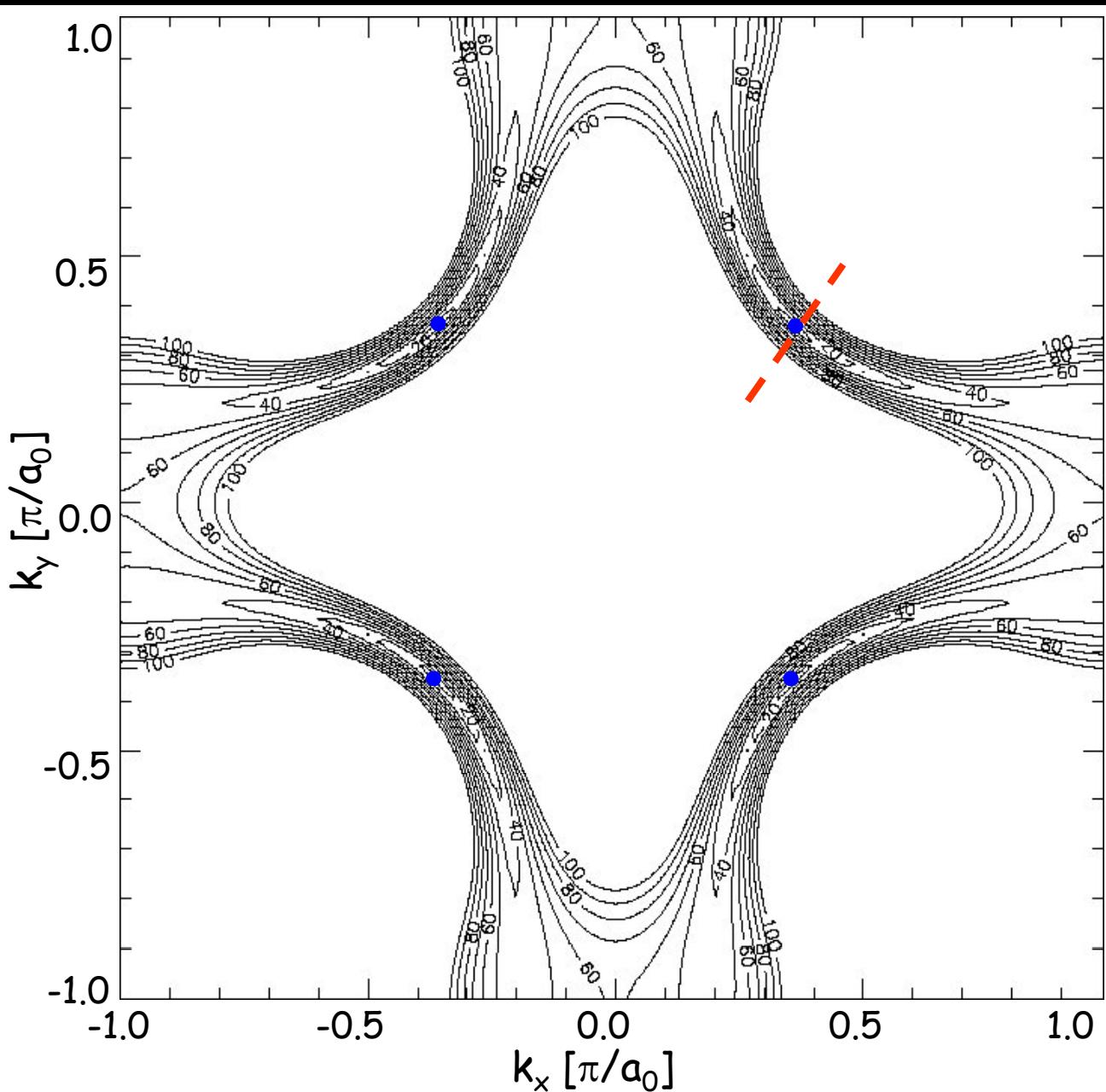


Campuzano *et al.*, PRL 64 2308 (1990)  
Dessau *et al.*, PRL 71 2781 (1993)  
Aebi *et al.*, PRL 72 2757 (1994)

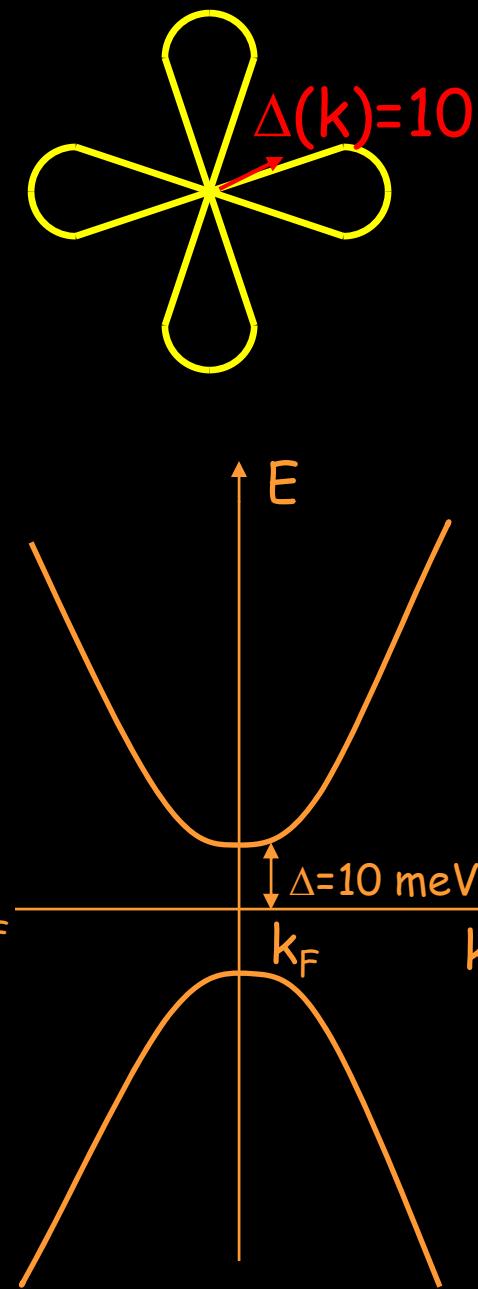
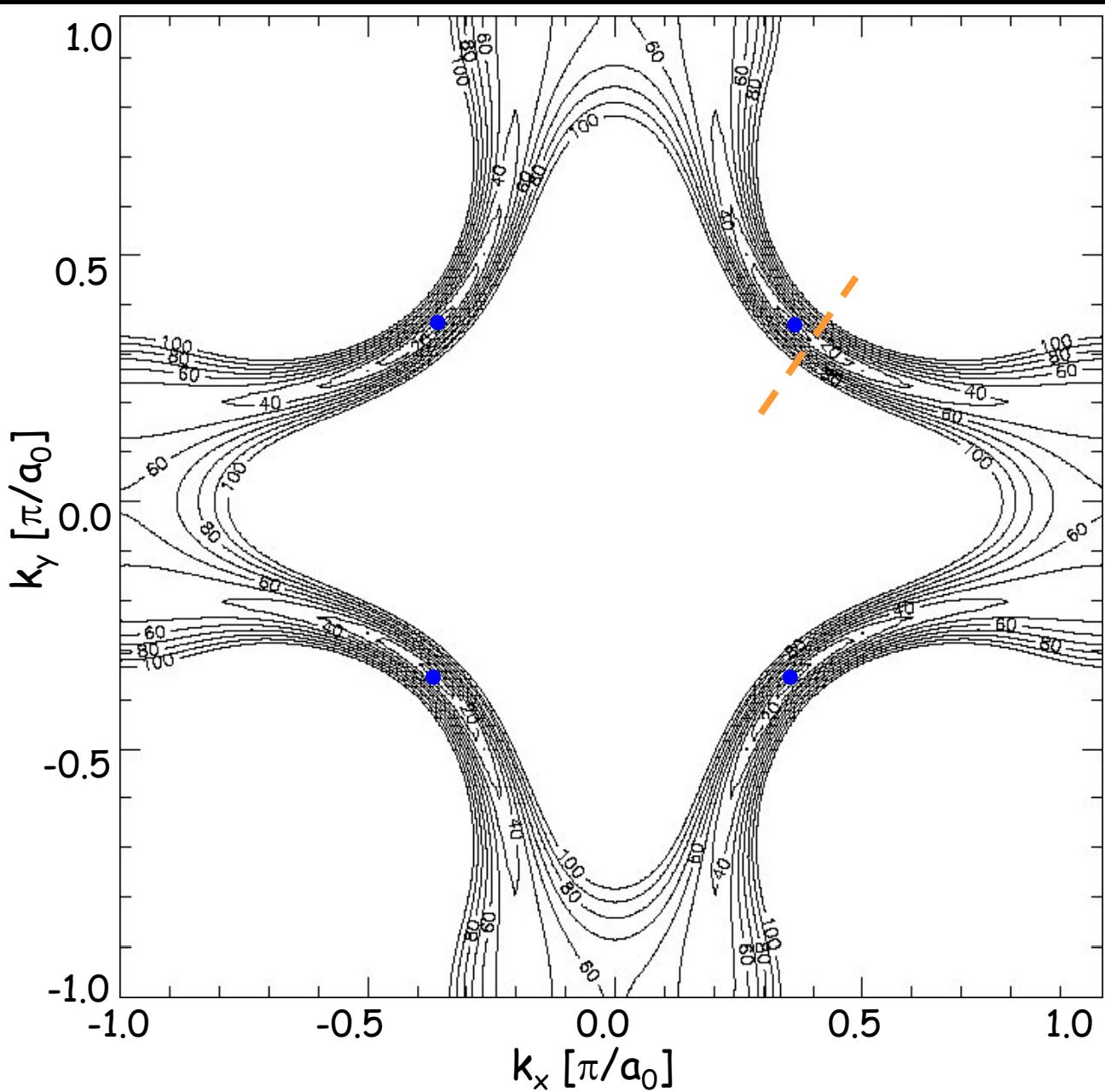
# ARPES: Normal State Fermi Surface & Band Structure



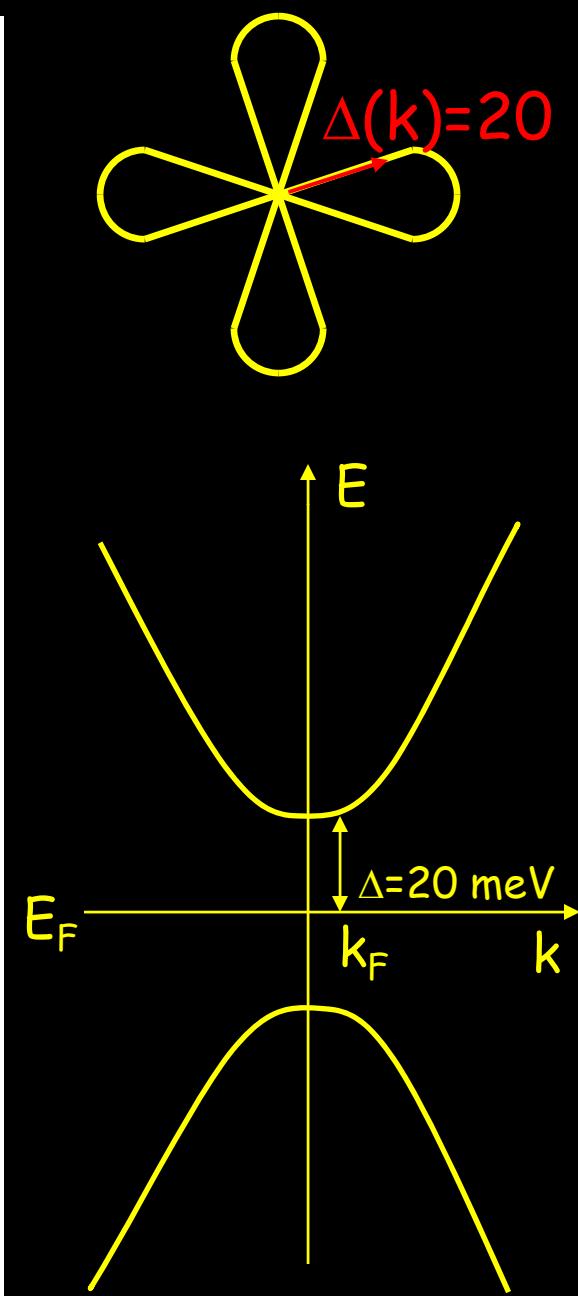
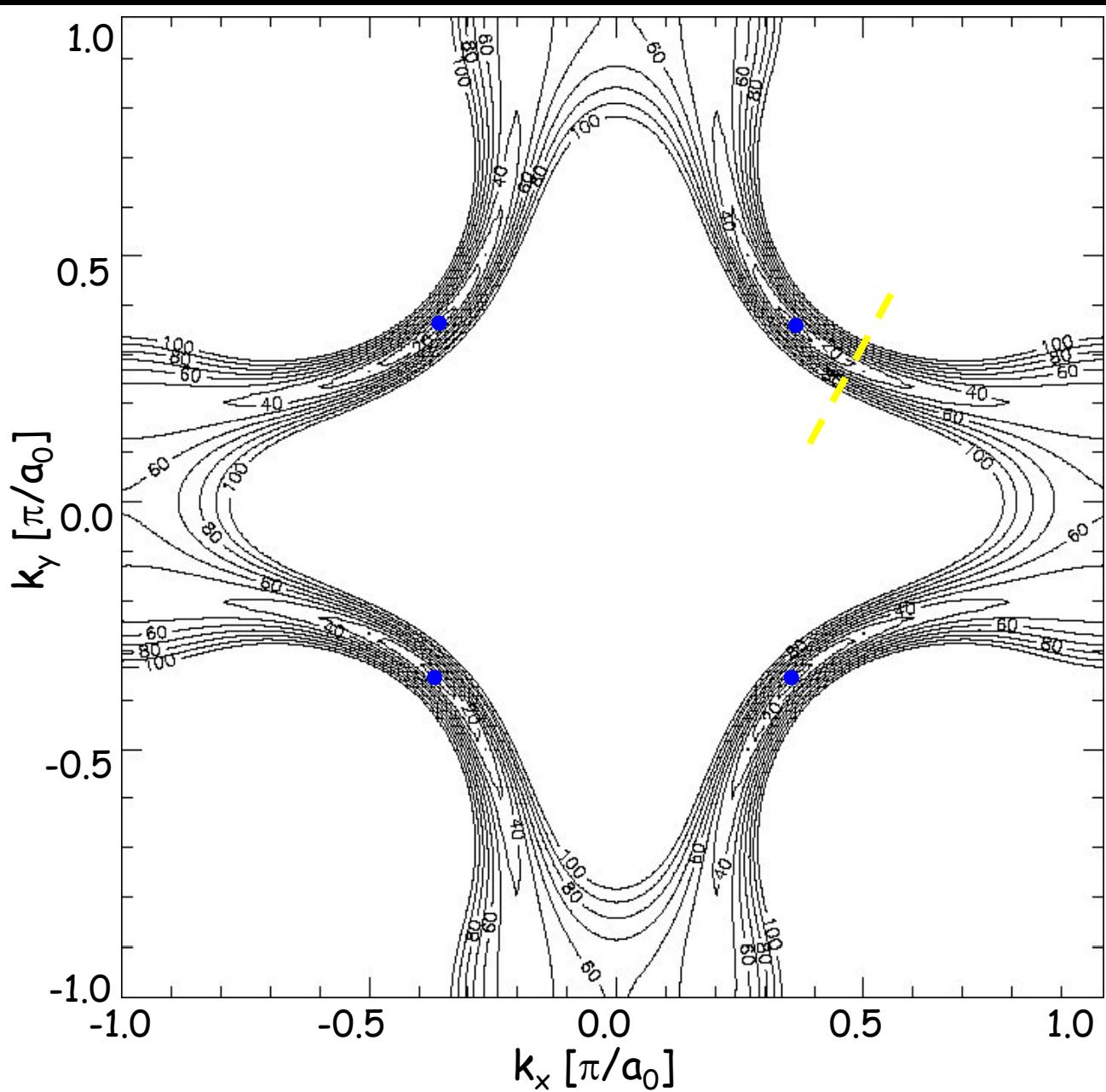
# ARPES: Superconducting anisotropic gap



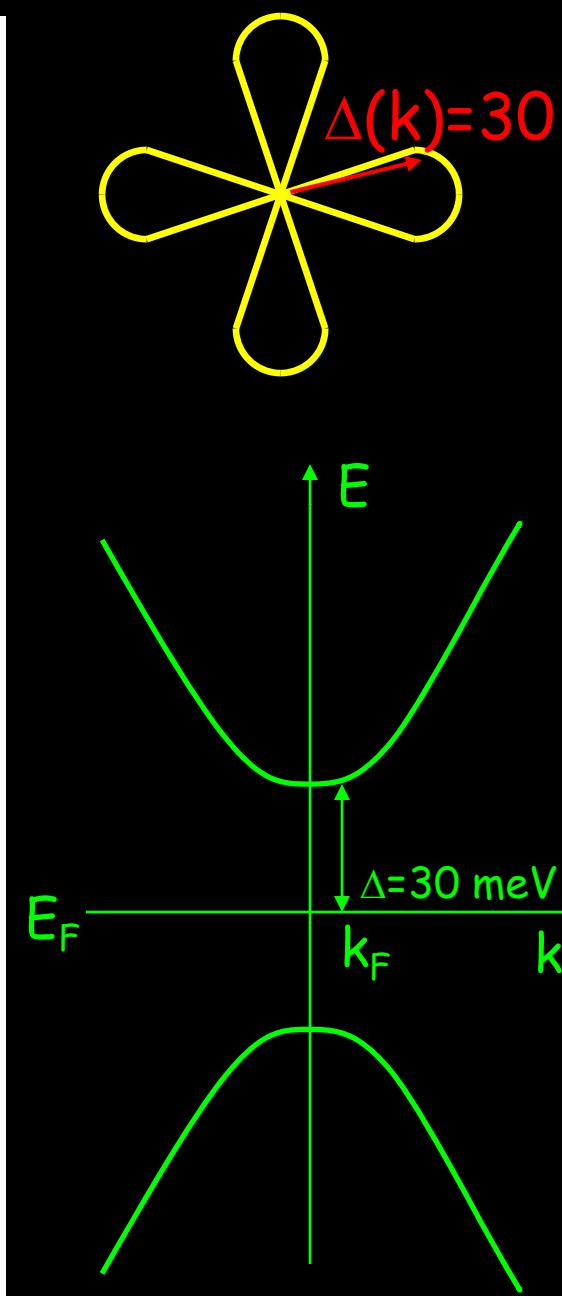
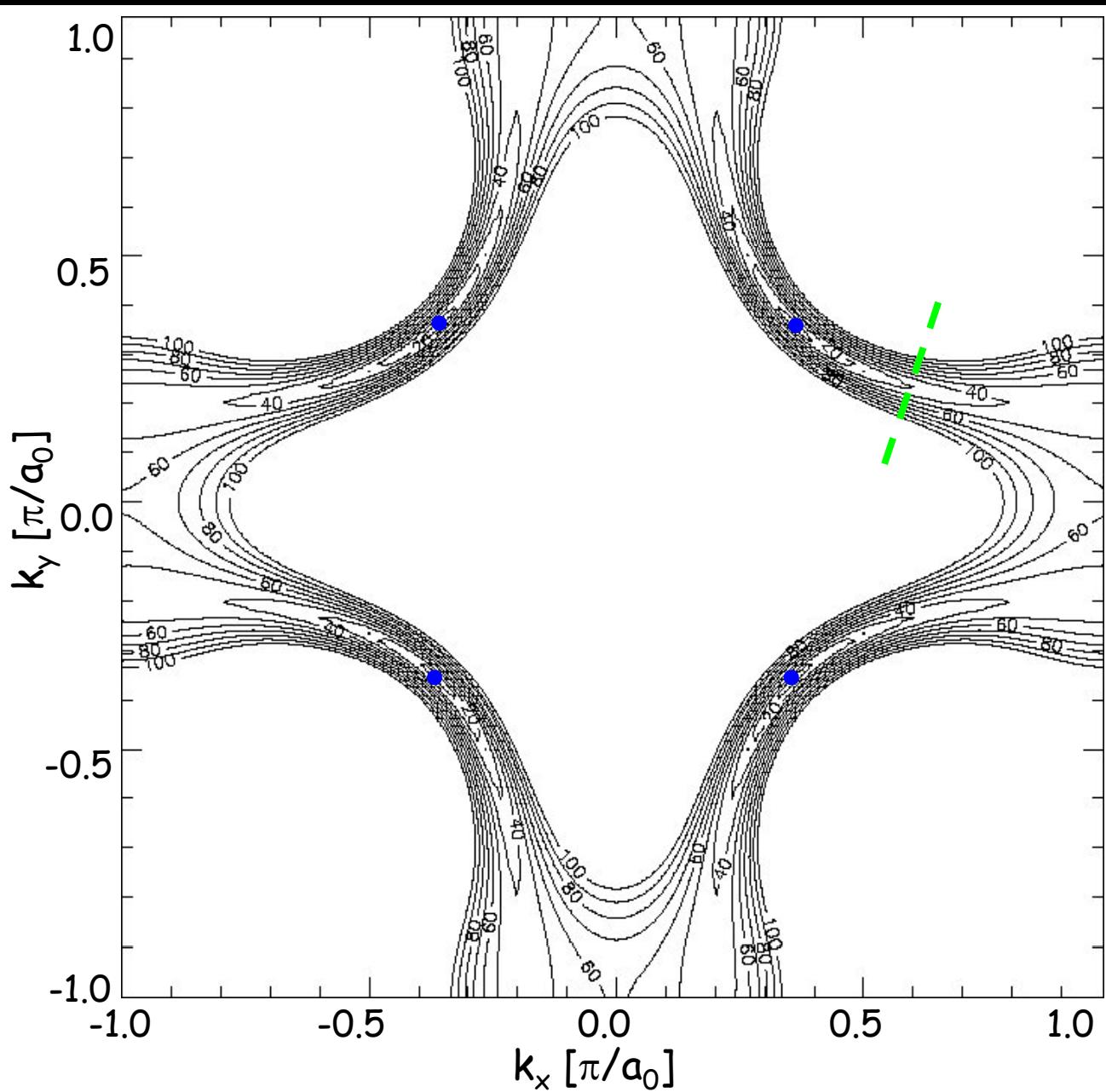
# ARPES: Superconducting anisotropic gap



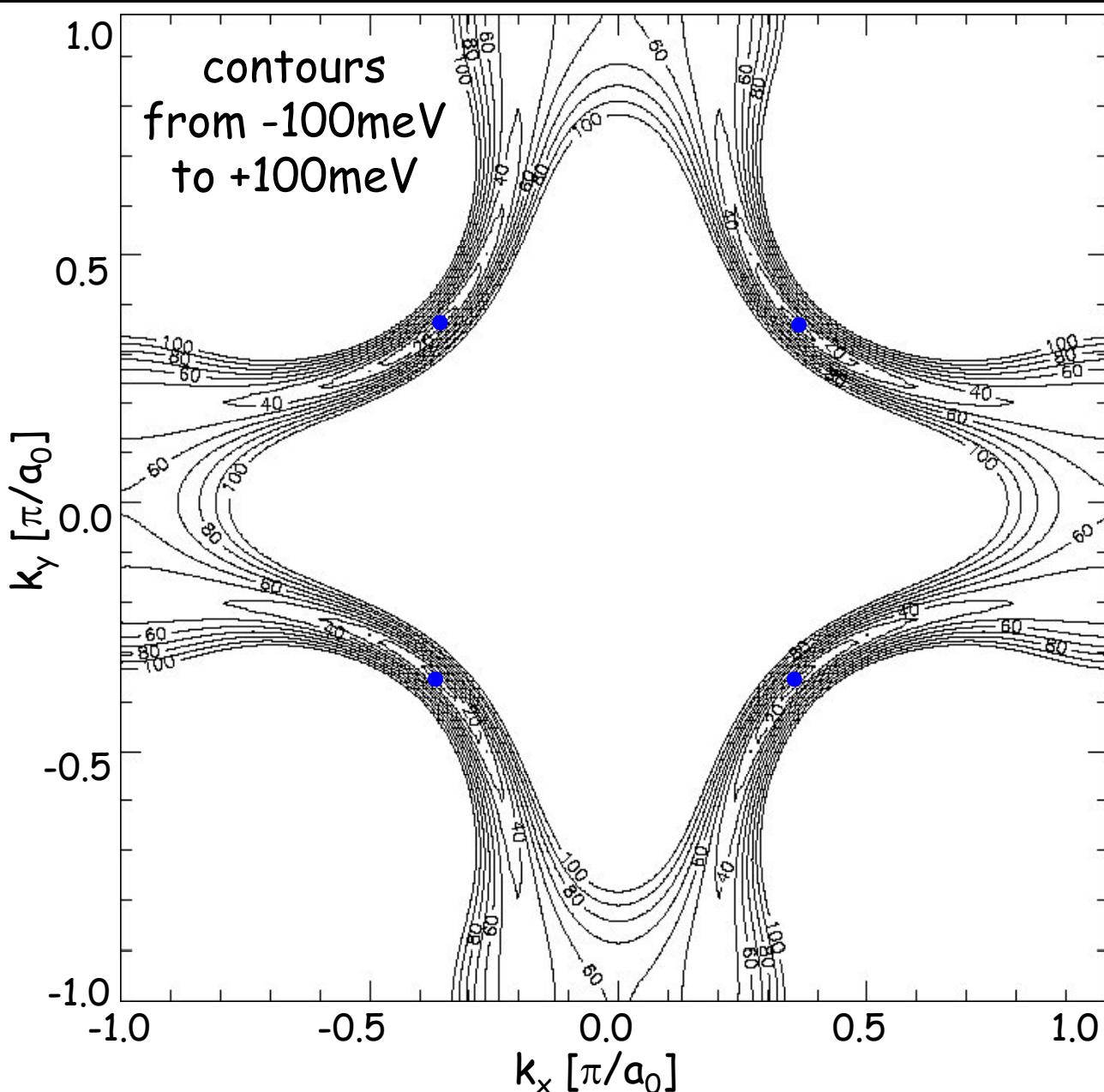
# ARPES: Superconducting anisotropic gap



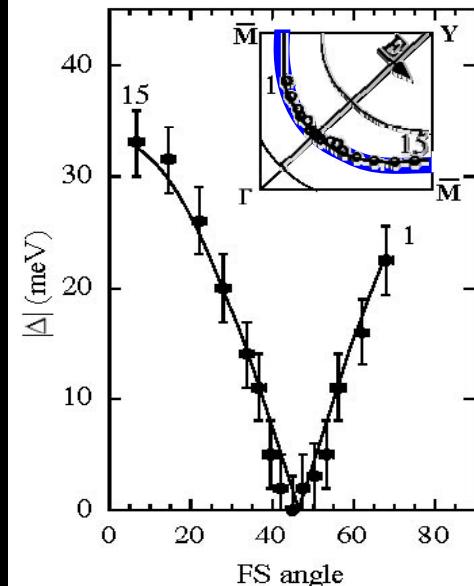
# ARPES: Superconducting anisotropic gap



# ARPES: Superconducting anisotropic gap

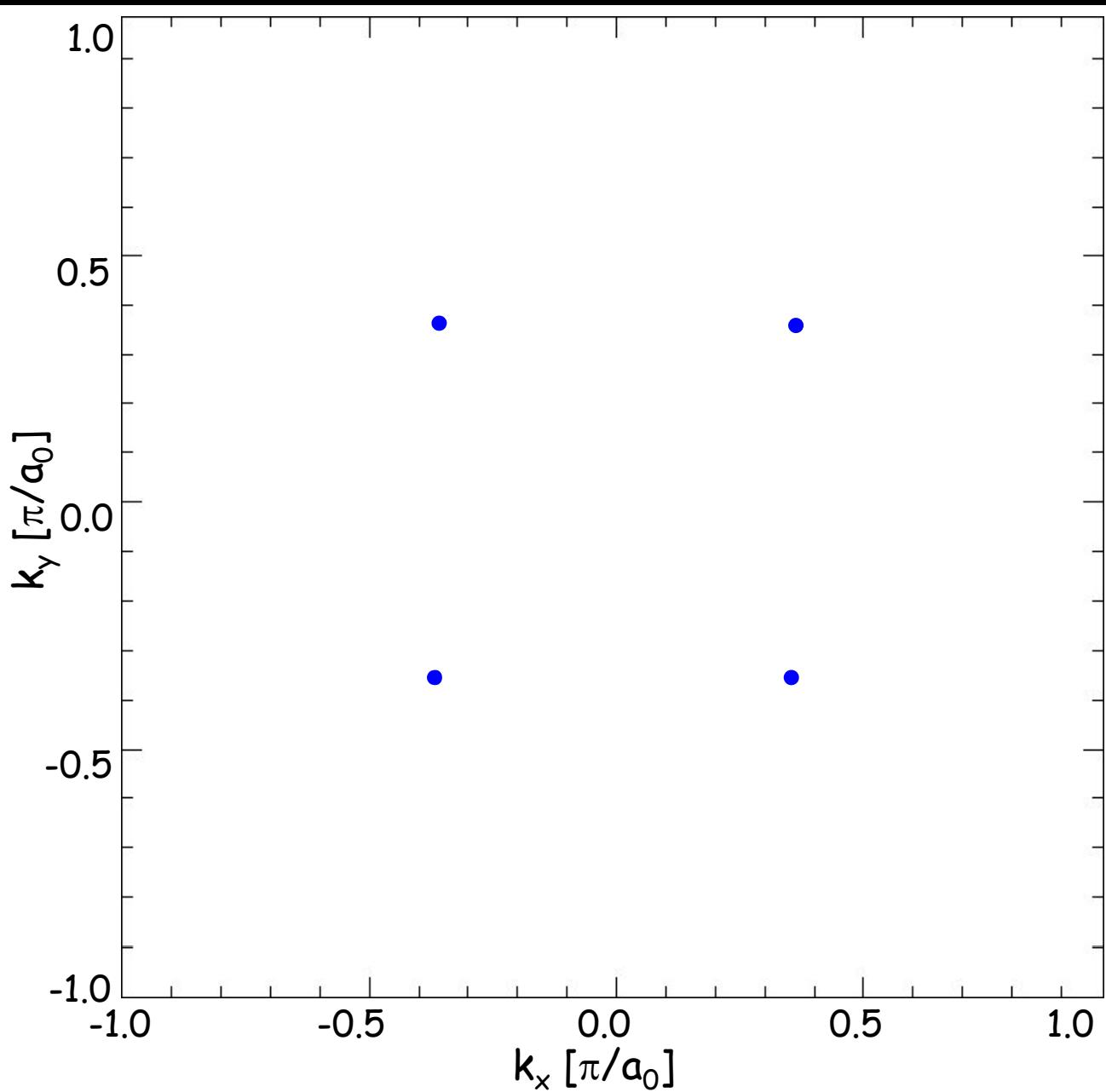


Ding *et al.*, PRLB 54, 9678 (1996)

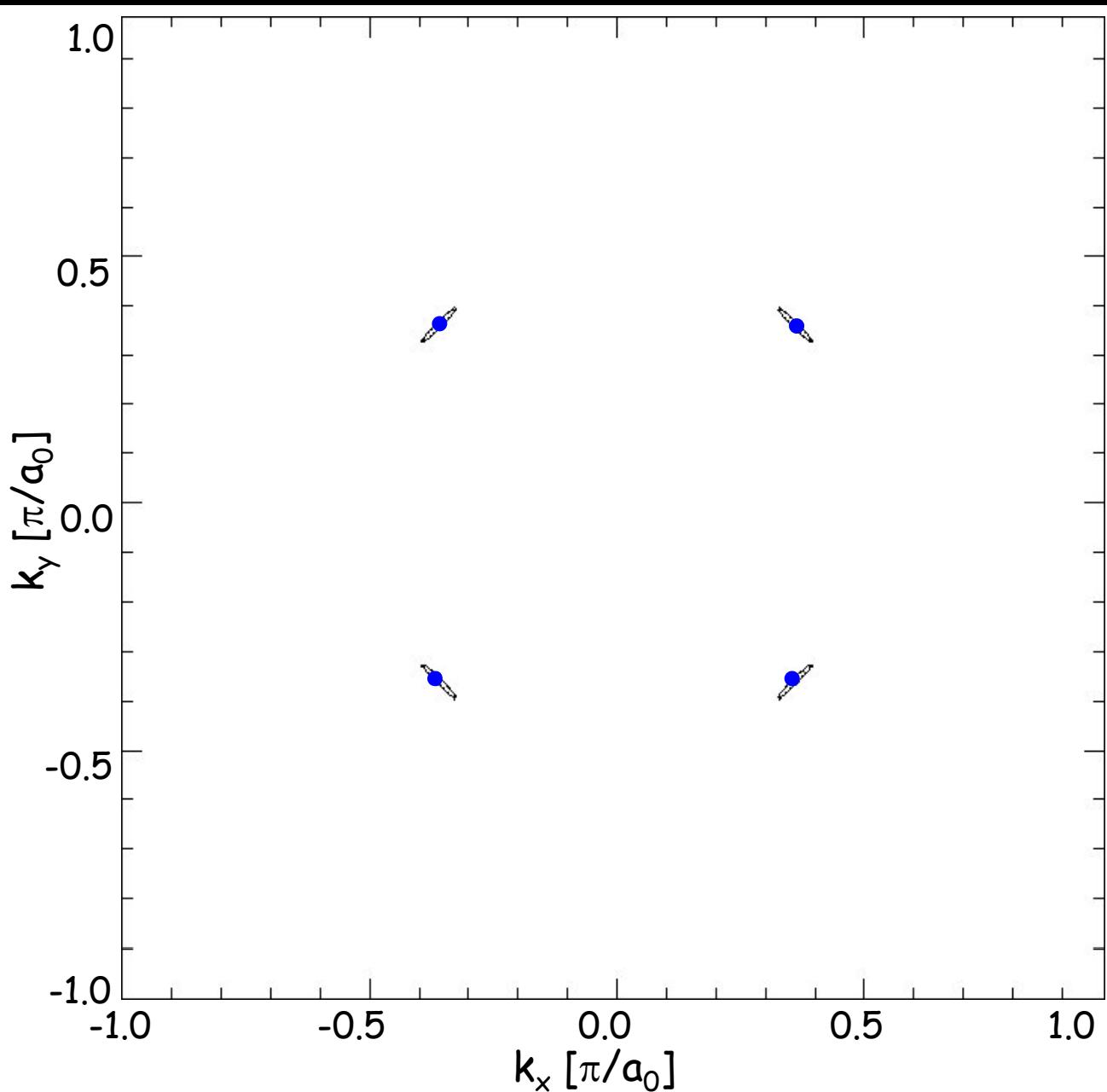


Shen *et al.*, PRL 70 1553 (1993)  
Ding *et al.*, PRB 54 9678 (1996)  
Mesot *et al.*, PRL 83 840 (1999)

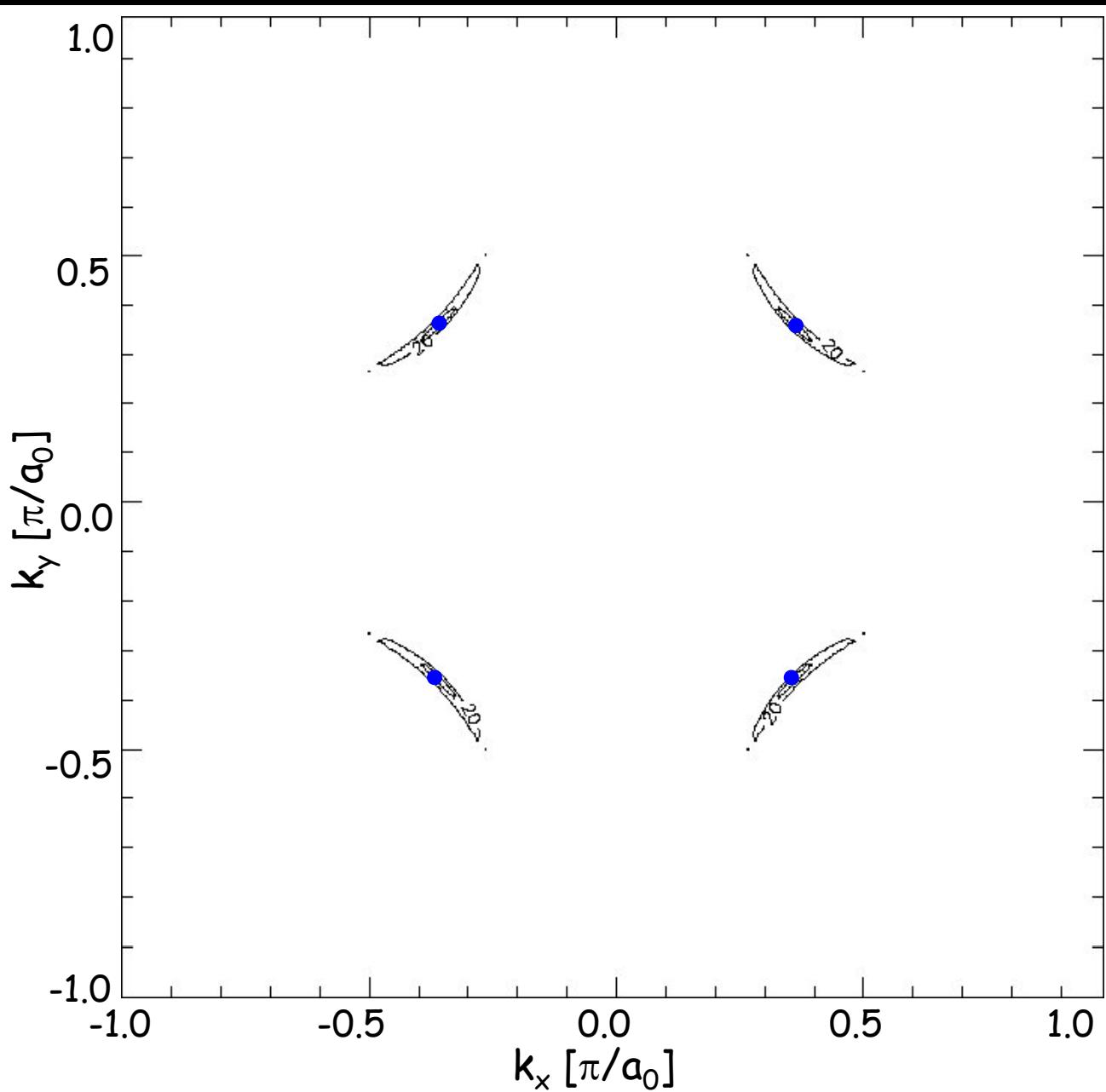
# 0 meV CCE: the Fermi points



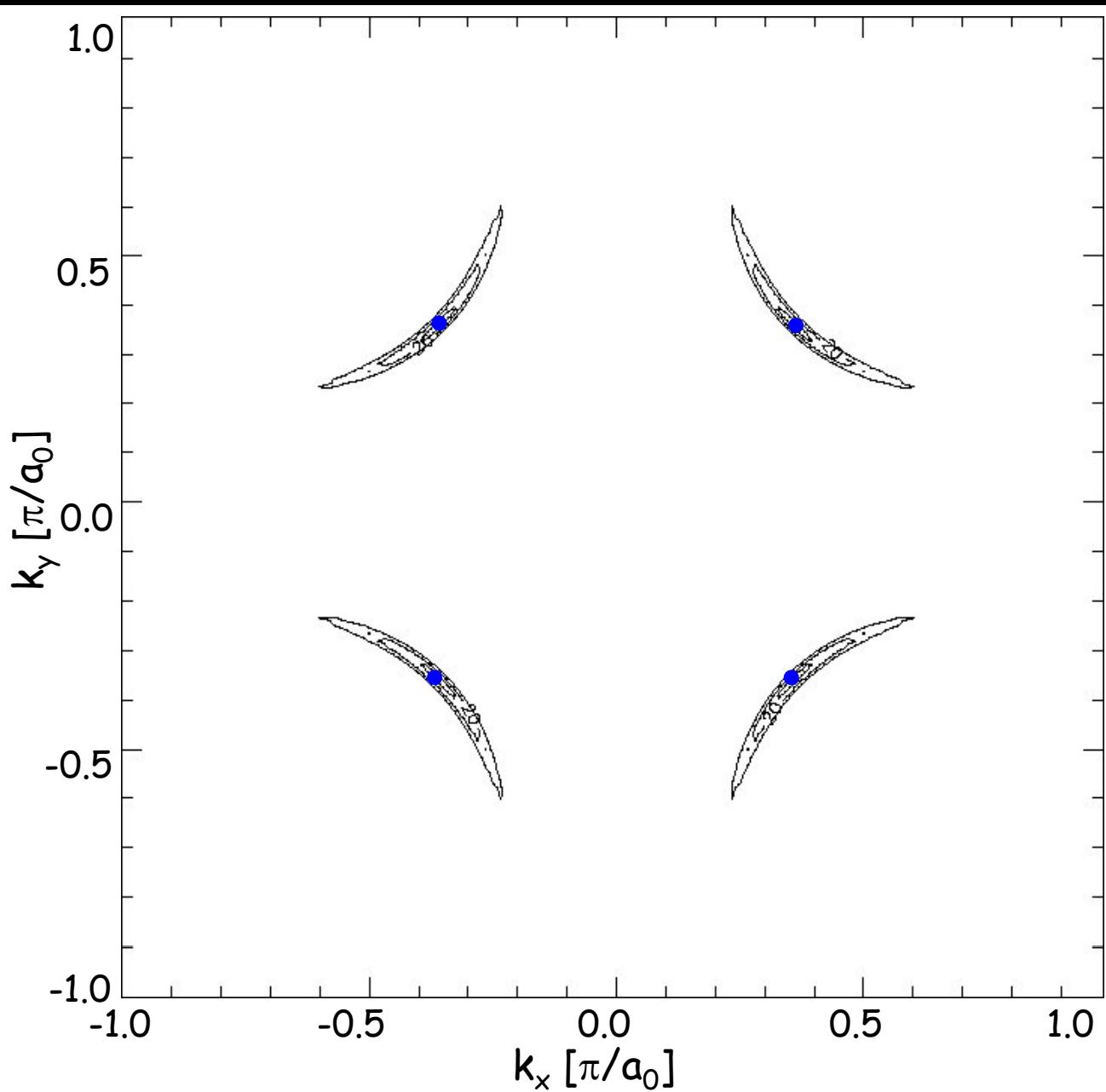
10 meV CCE



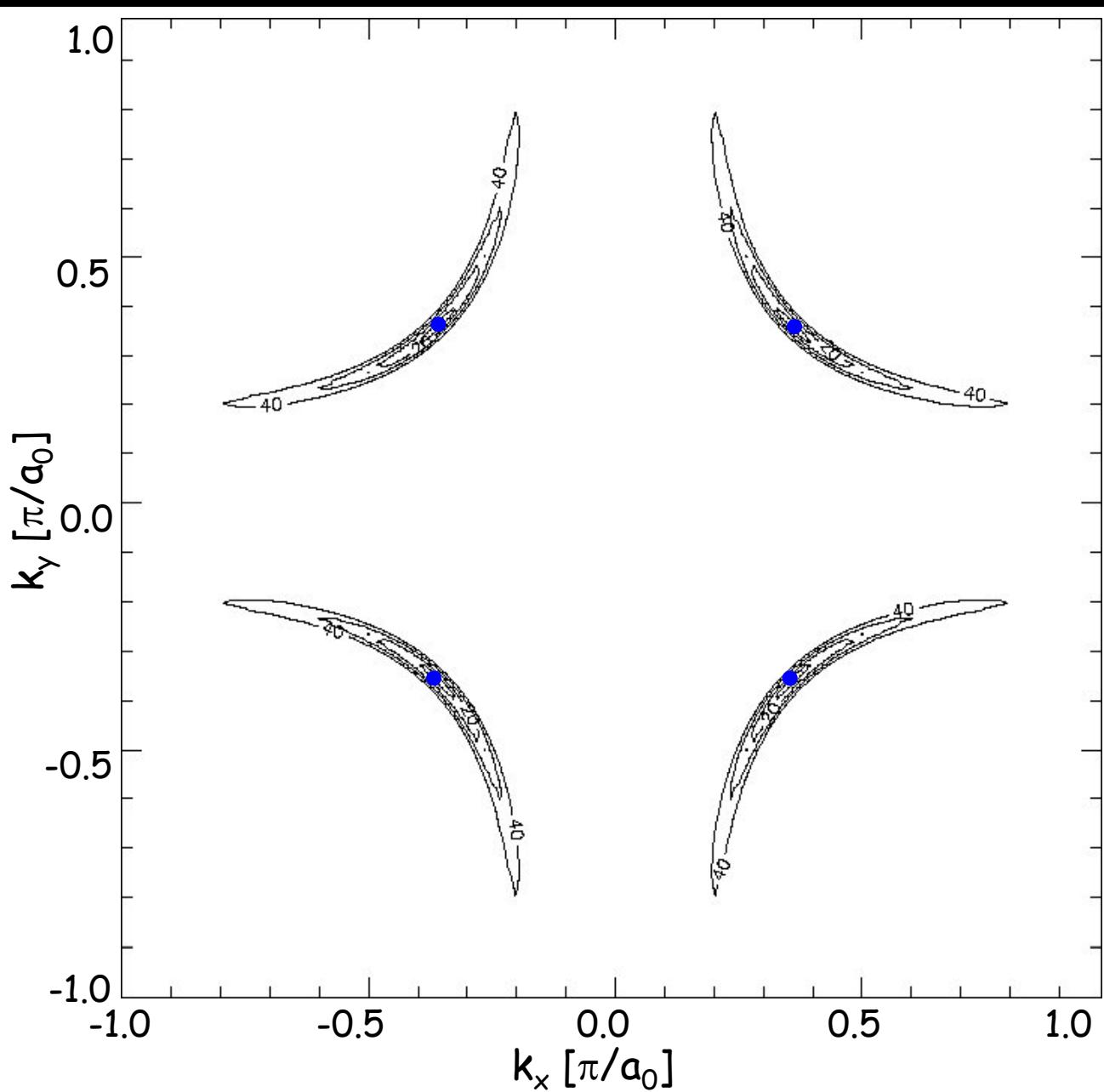
# 20 meV CCE



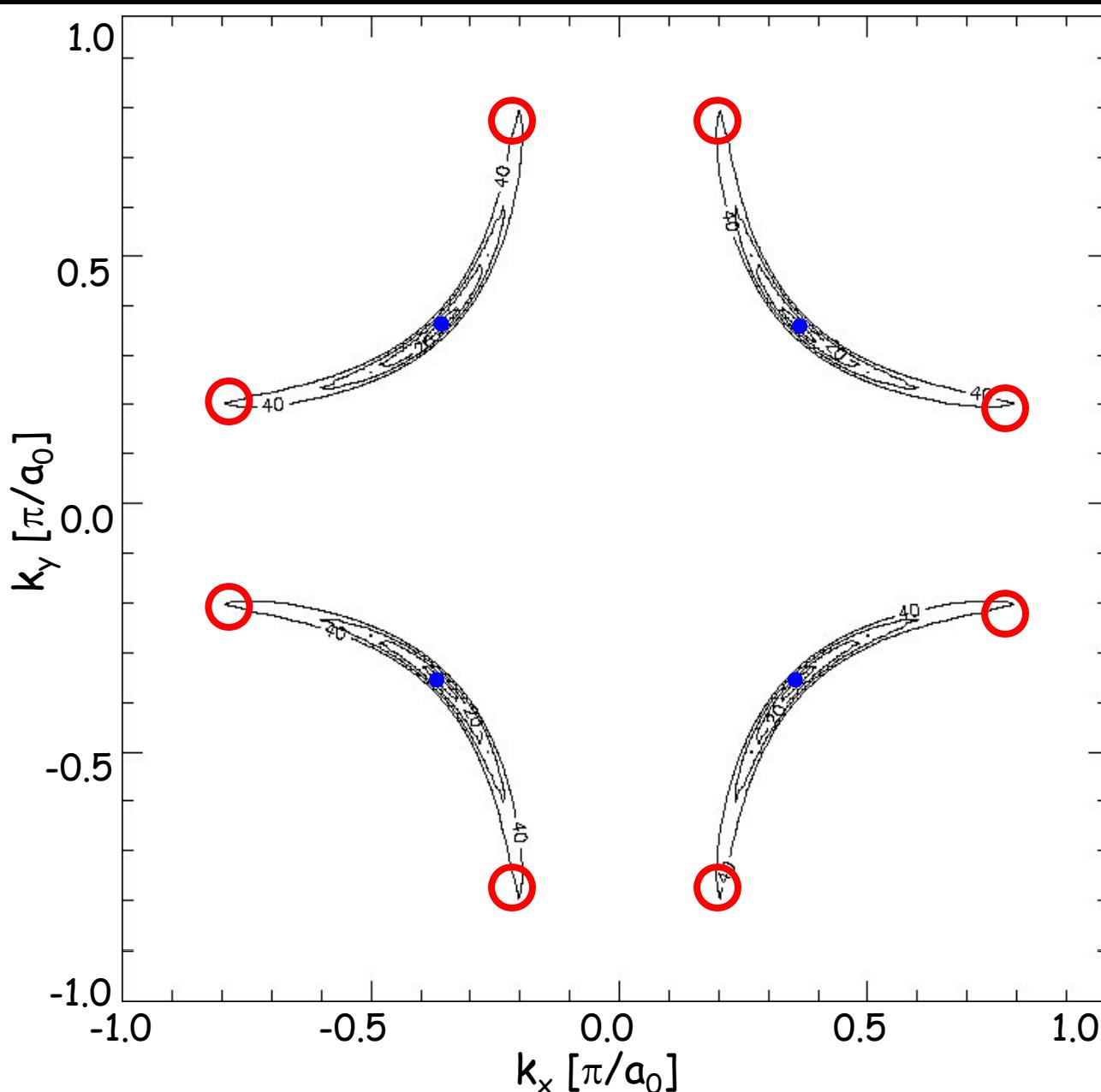
30 meV CCE



# 40 meV CCE



Octet of regions at ends of 'bananas' have largest  $|dk|/dE$



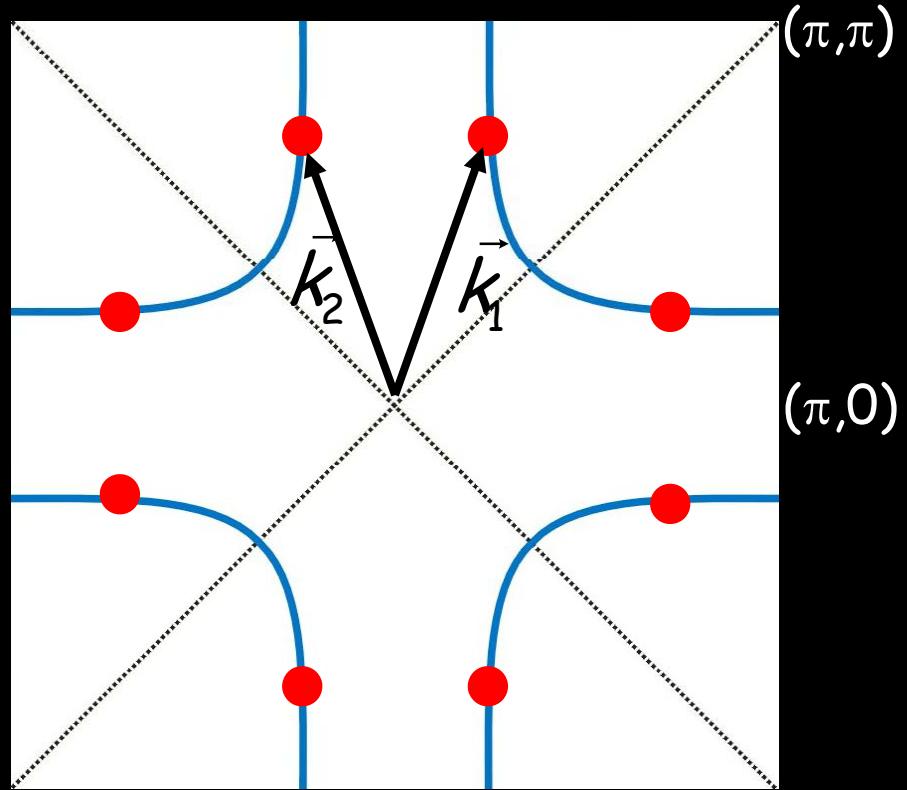
Density of States

$$n(E) = \oint_{E(k)=E} \frac{1}{|\nabla_k E(\vec{k})|} dk$$



The octet of  $k$ -space locations at the tips of the 'bananas' provide maximum contribution to  $n_{i,f}(E)$  and thus dominate elastic scattering processes.

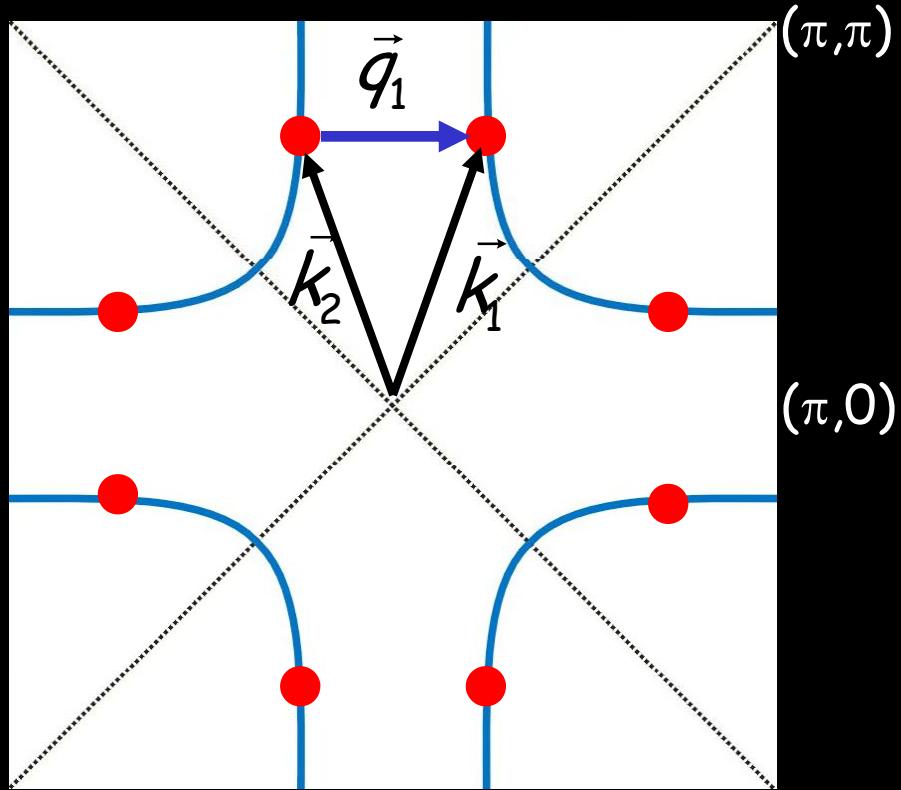
# The scattering vectors of QI model



*k*-space:

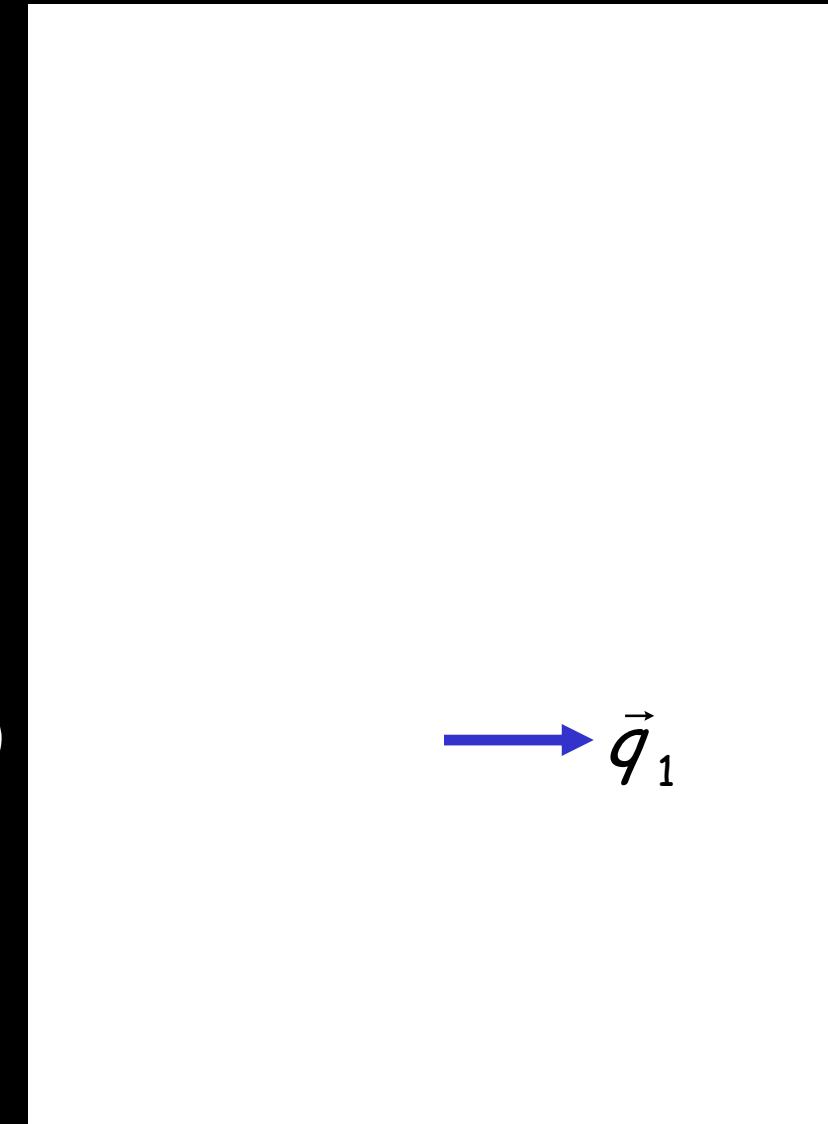
- unperturbed eigenstates
- not directly accessible to STM
- measured by ARPES

# The scattering vectors of QI model



$k$ -space:

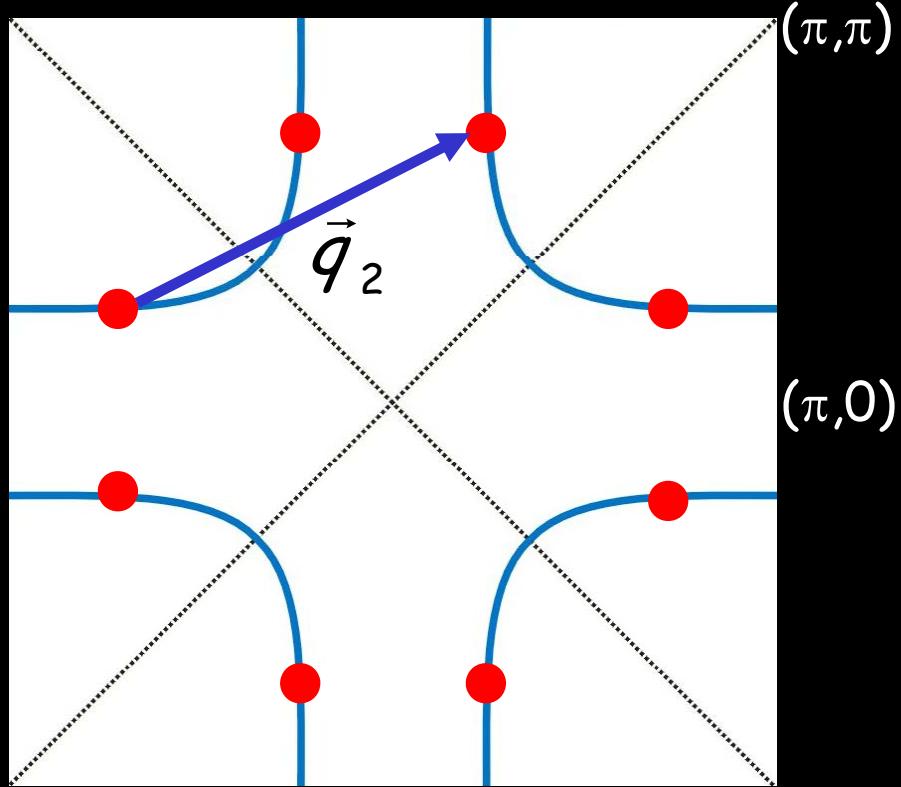
- unperturbed eigenstates
- not directly accessible to STM
- measured by ARPES



$q$ -space:

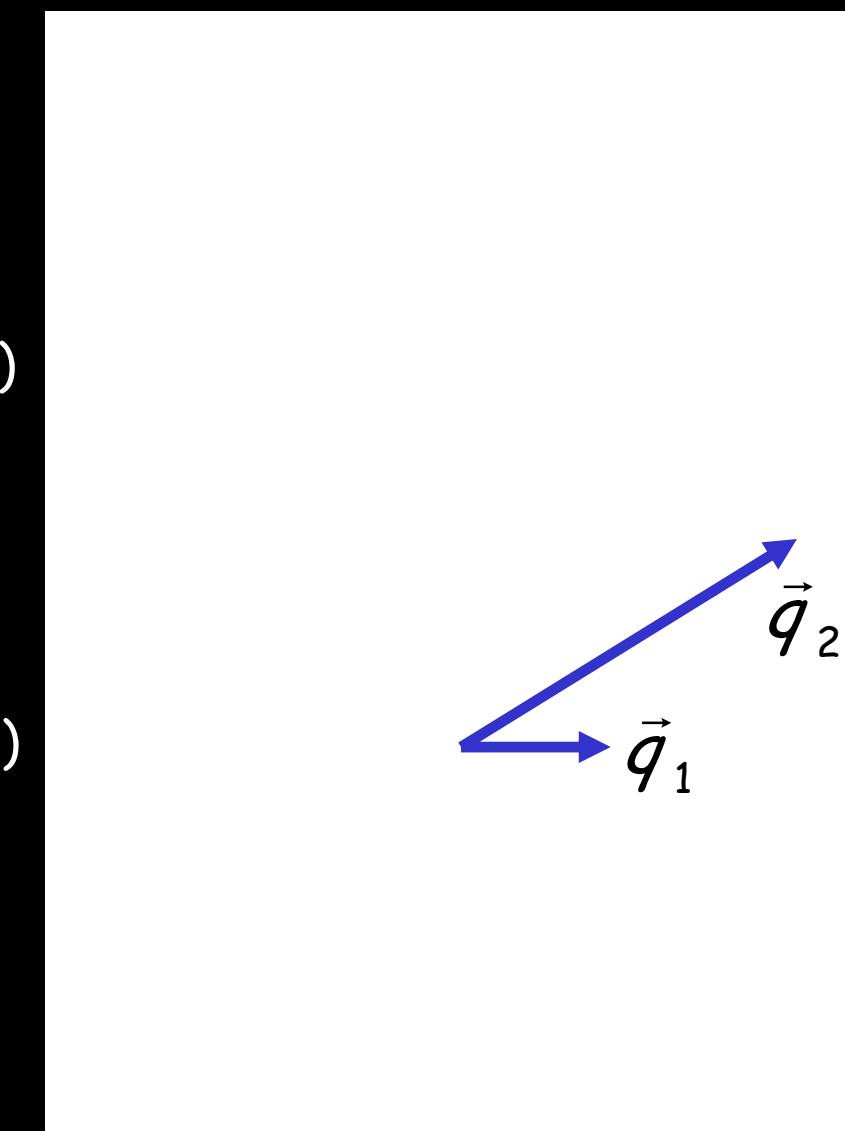
- Scattering  $\rightarrow$  standing waves  $q = 2\pi/\lambda$
- Measure  $q$  from FT of LDOS image

# The scattering vectors of QI model



$k$ -space:

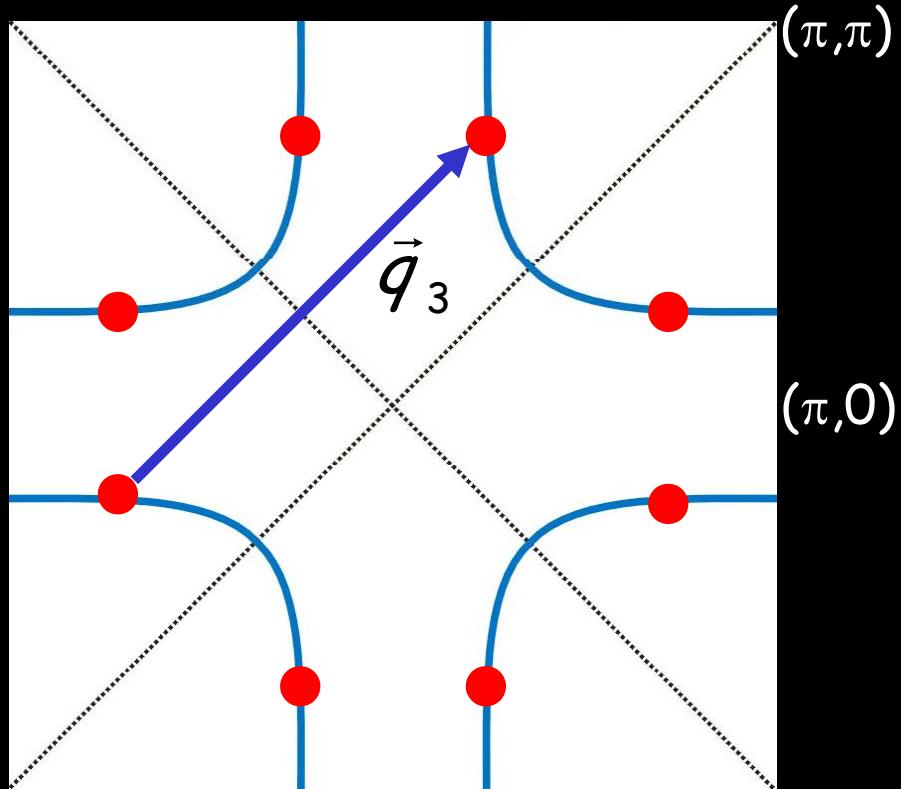
- unperturbed eigenstates
- not directly accessible to STM
- measured by ARPES



$q$ -space:

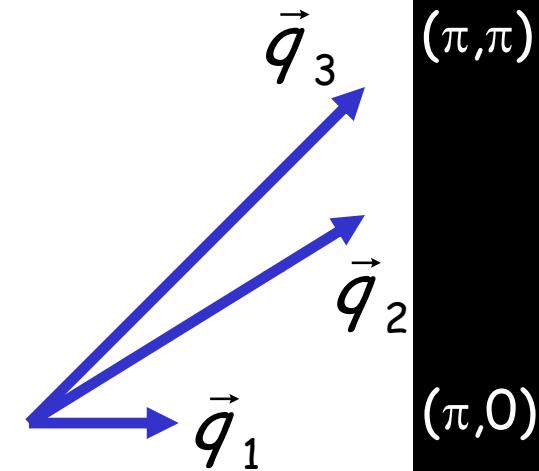
- Scattering  $\rightarrow$  standing waves  $q = 2\pi/\lambda$
- Measure  $q$  from FT of LDOS image

# The scattering vectors of QI model



$k$ -space:

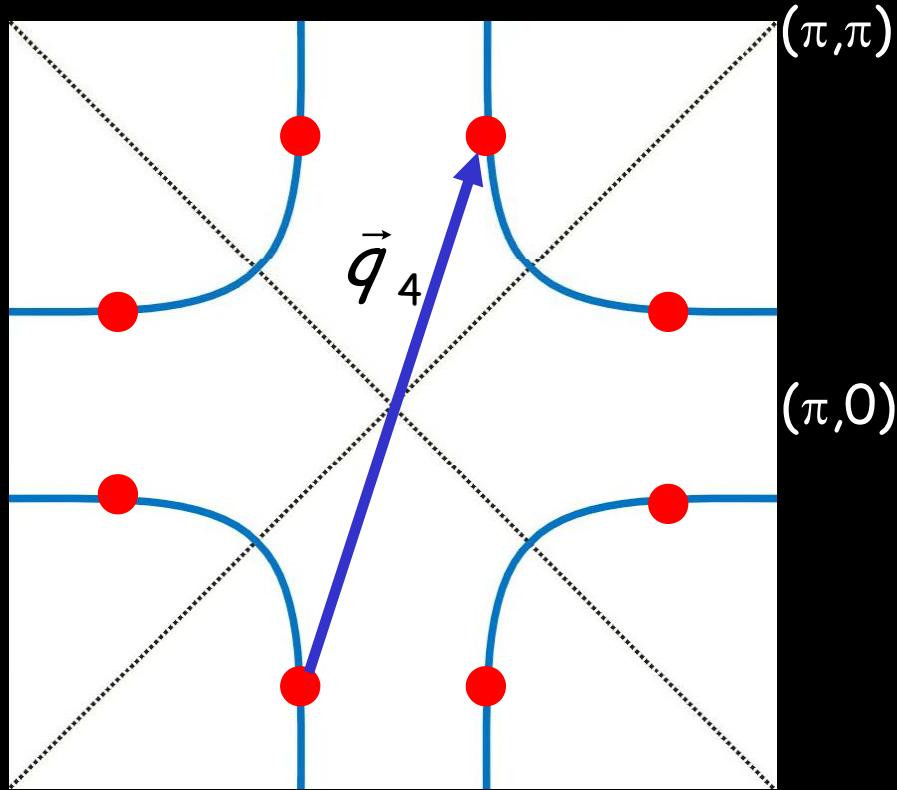
- unperturbed eigenstates
- not directly accessible to STM
- measured by ARPES



$q$ -space:

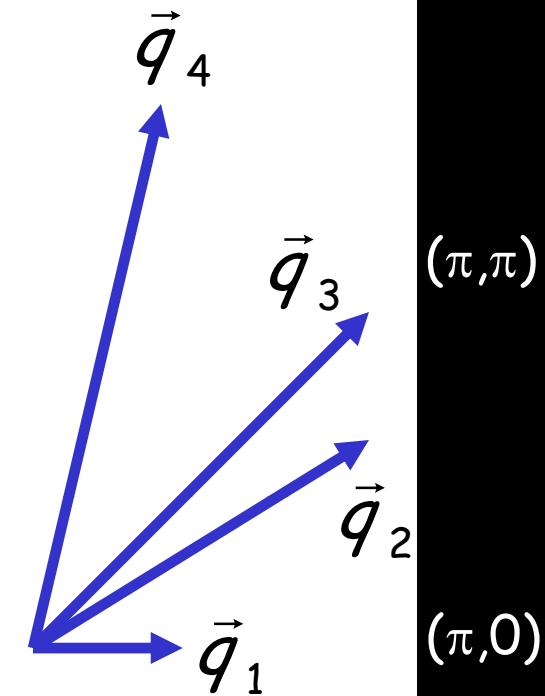
- Scattering  $\rightarrow$  standing waves  $q = 2\pi/\lambda$
- Measure  $q$  from FT of LDOS image

# The scattering vectors of QI model



$k$ -space:

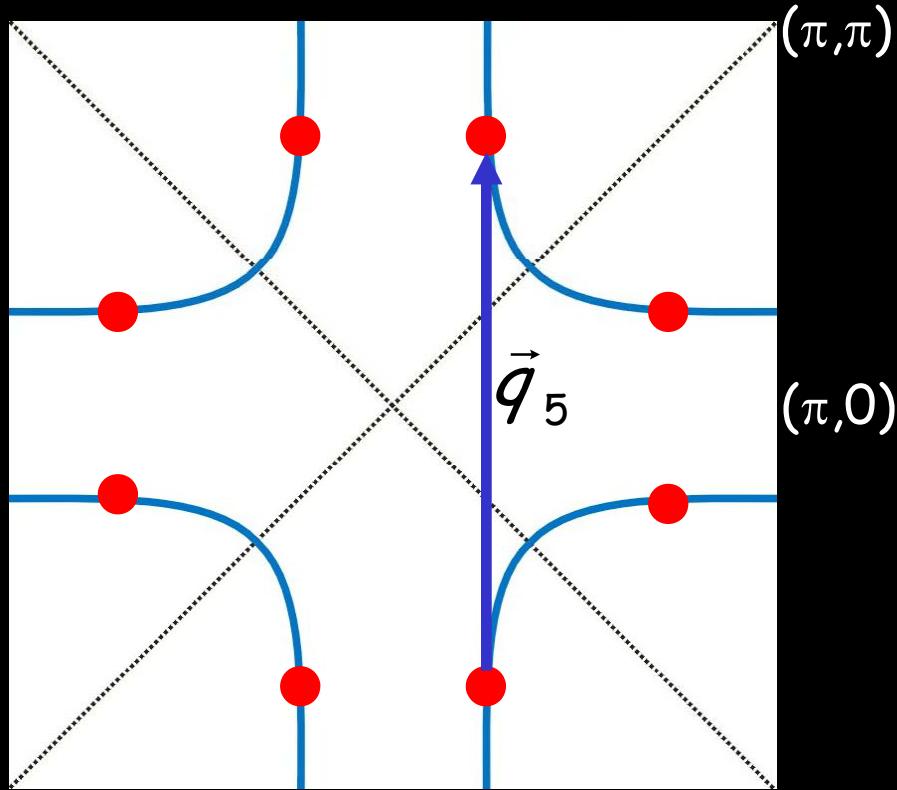
- unperturbed eigenstates
- not directly accessible to STM
- measured by ARPES



$q$ -space:

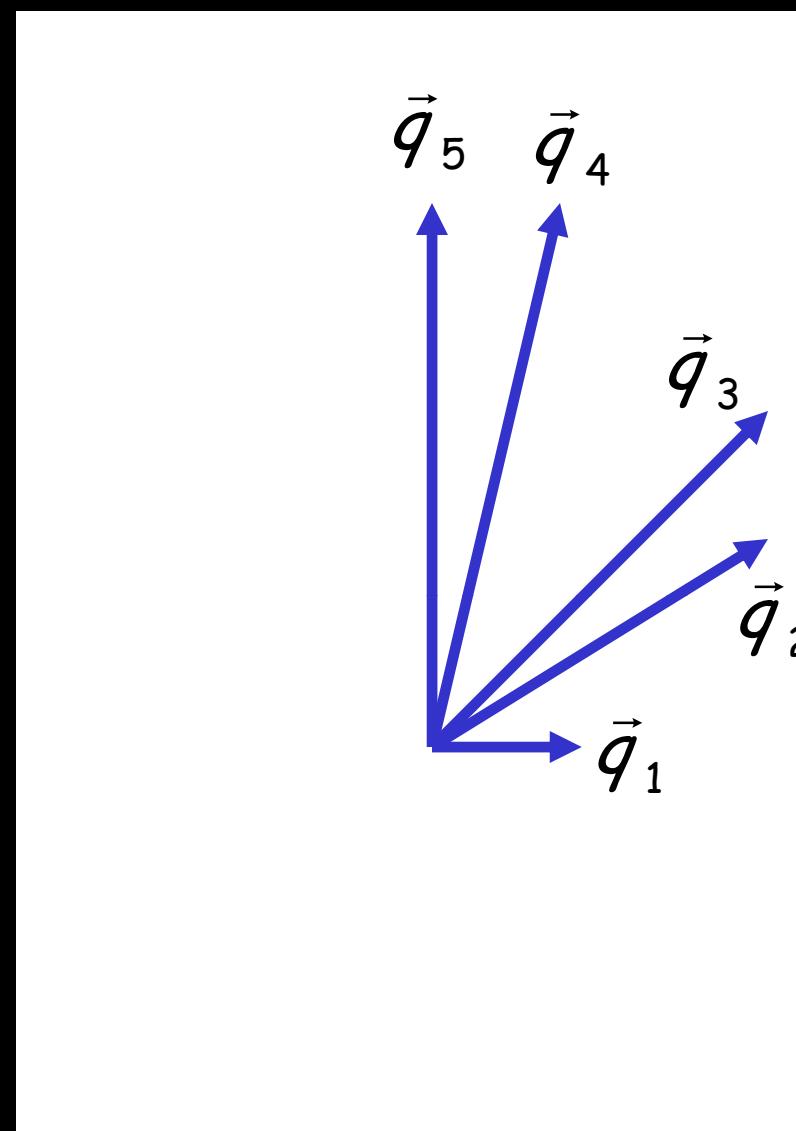
- Scattering  $\rightarrow$  standing waves  $q = 2\pi/\lambda$
- Measure  $q$  from FT of LDOS image

# The scattering vectors of QI model



$k$ -space:

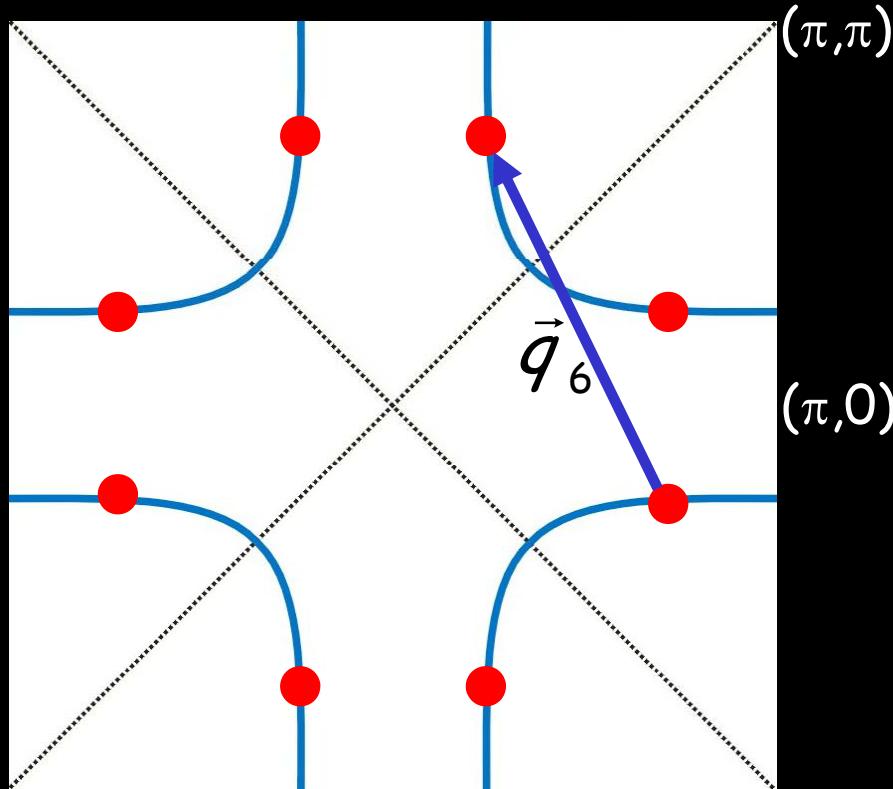
- unperturbed eigenstates
- not directly accessible to STM
- measured by ARPES



$q$ -space:

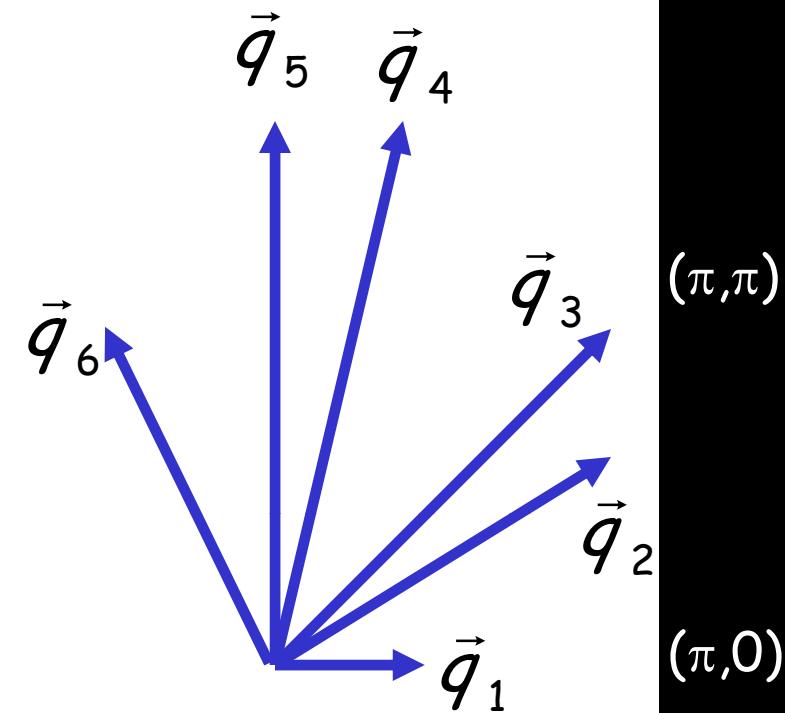
- Scattering  $\rightarrow$  standing waves  $q = 2\pi/\lambda$
- Measure  $q$  from FT of LDOS image

# The scattering vectors of QI model



$k$ -space:

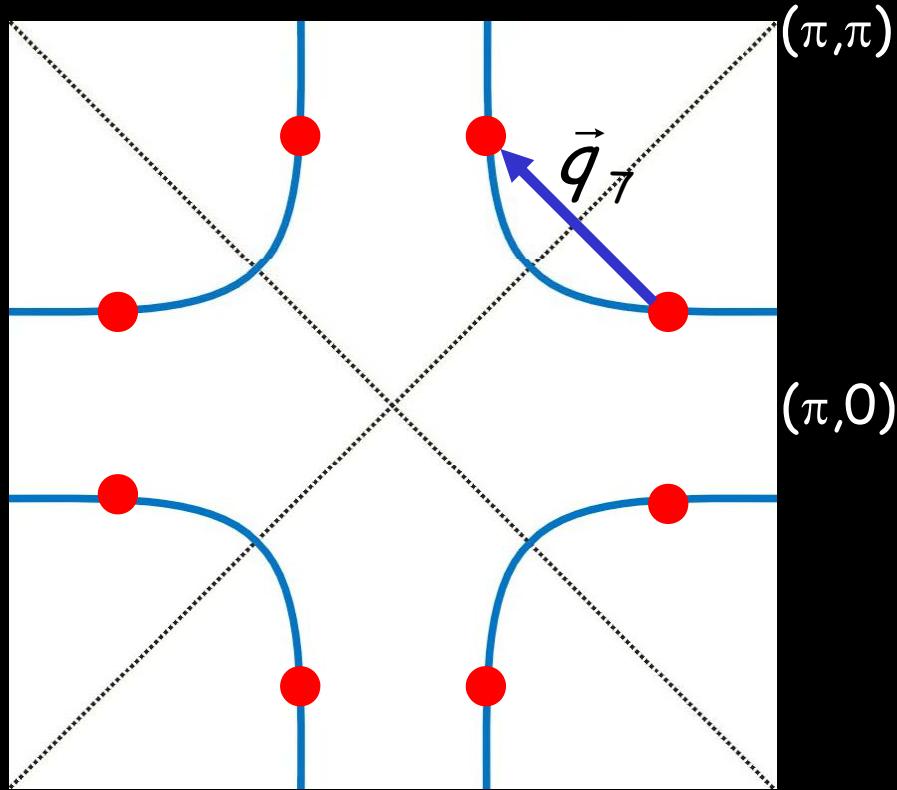
- unperturbed eigenstates
- not directly accessible to STM
- measured by ARPES



$q$ -space:

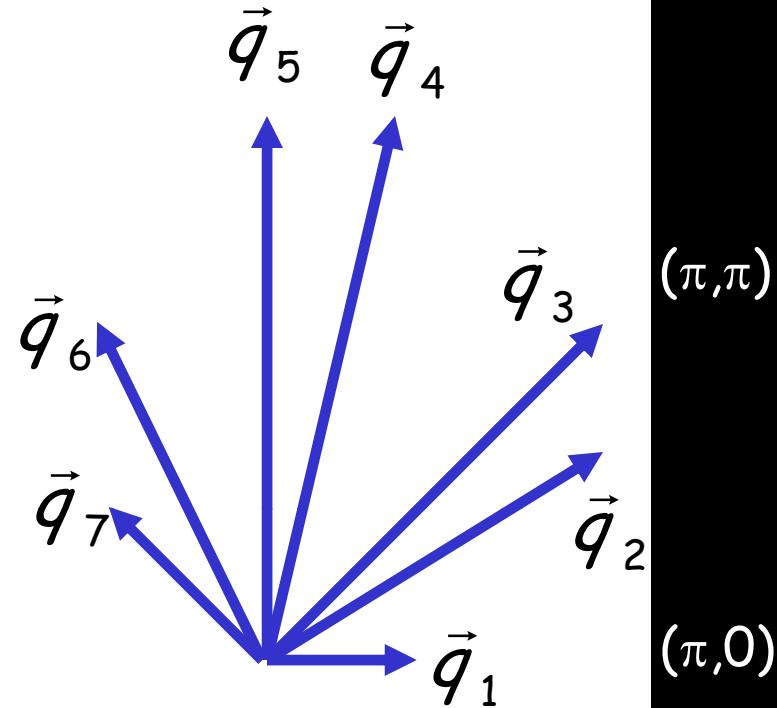
- Scattering  $\rightarrow$  standing waves  $q = 2\pi/\lambda$
- Measure  $q$  from FT of LDOS image

# The scattering vectors of QI model



*k*-space:

- unperturbed eigenstates
- not directly accessible to STM
- measured by ARPES



Total sets of  $q_i$  ( $7 \times 8$ ) : 56

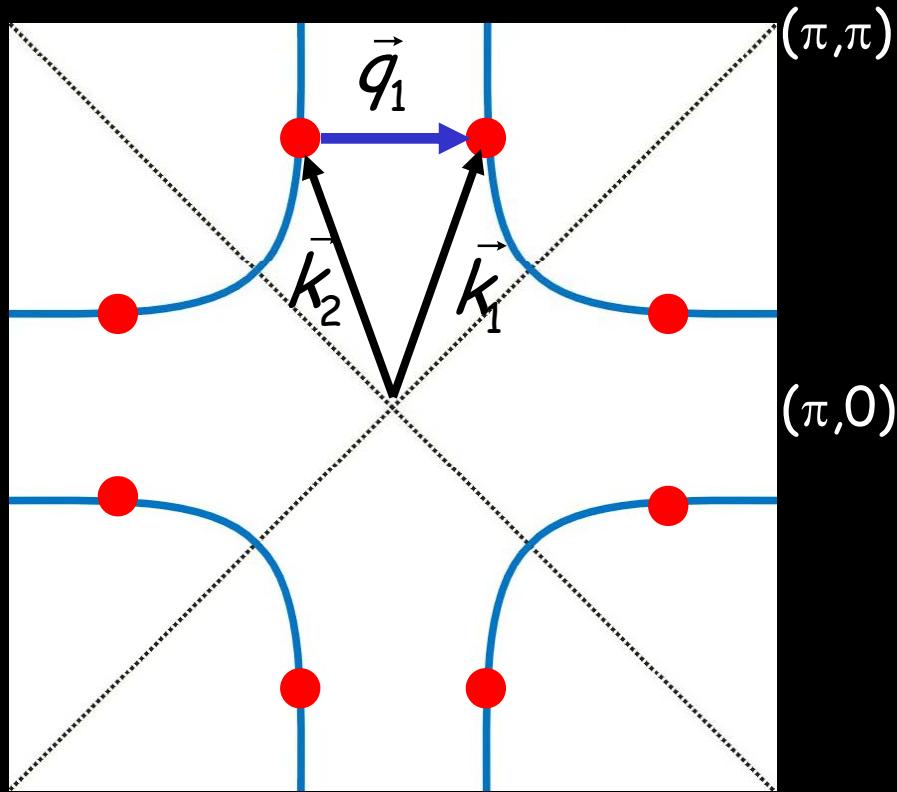
Inequivalent sets of  $q_i$  : 32

Distinguishable via FT-STS : 16

*q*-space:

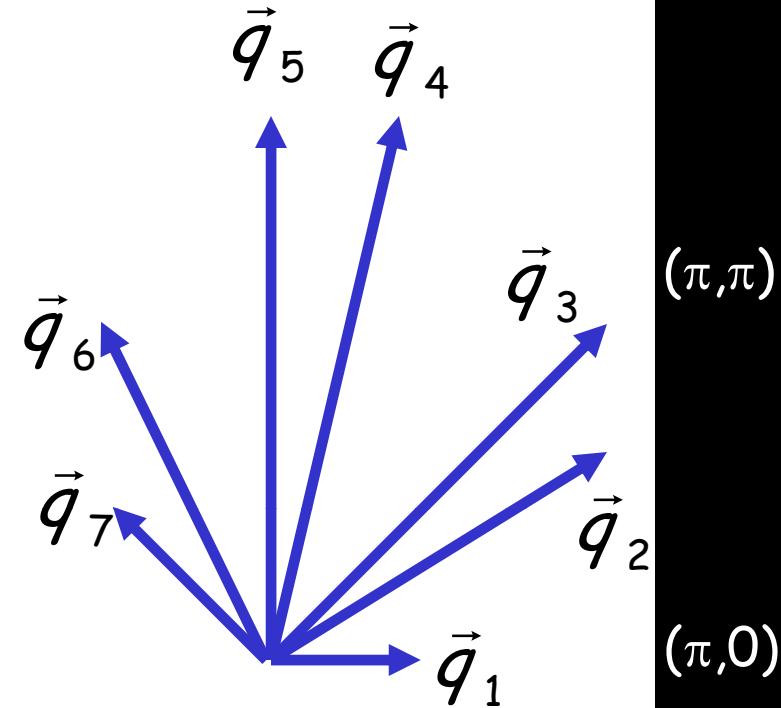
- Scattering → standing waves  $q = 2\pi/\lambda$
- Measure  $q$  from FT of LDOS image

# The scattering vectors of QI model



$k$ -space:

- unperturbed eigenstates
- not directly accessible to STM
- measured by ARPES



Total sets of  $q_i$  ( $7 \times 8$ ) : 56

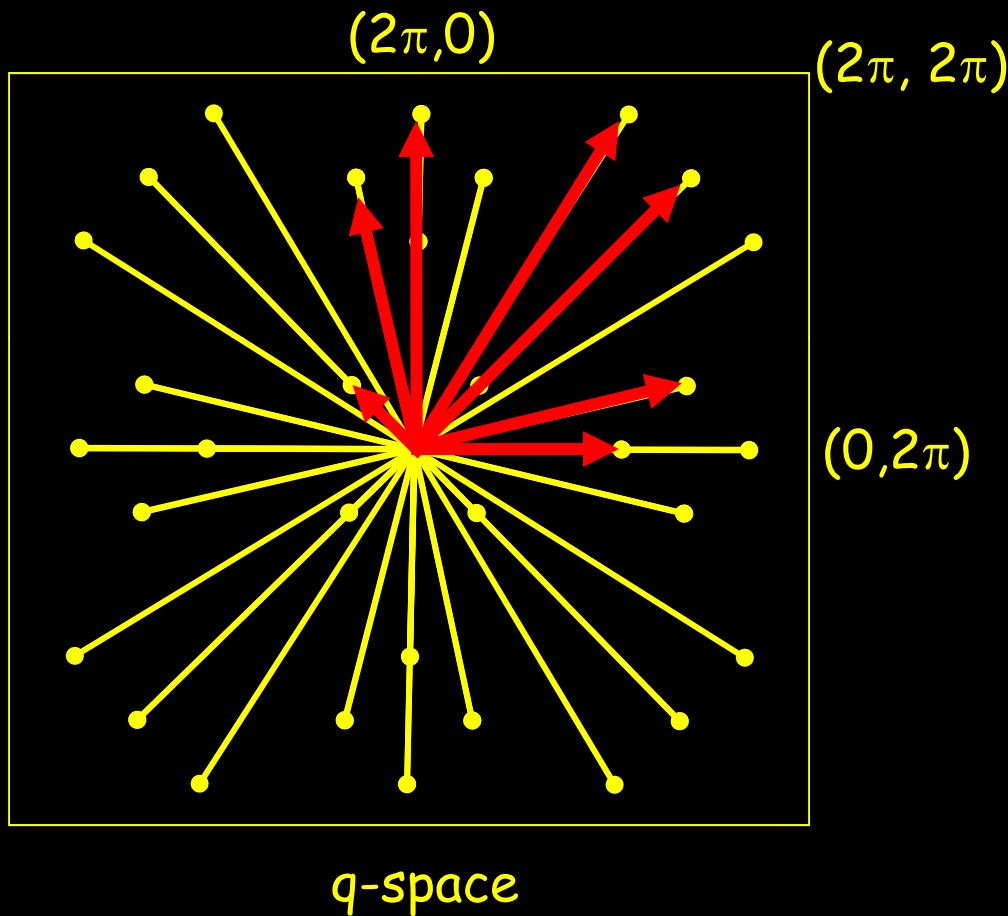
Inequivalent sets of  $q_i$  : 32

Distinguishable via FT-STS : 16

$q$ -space:

- Scattering  $\rightarrow$  standing waves  $q = 2\pi/\lambda$
- Measure  $q$  from FT of LDOS image

# Expected structure of FFT of LDOS( $r, E$ ) (for a fixed $E$ )

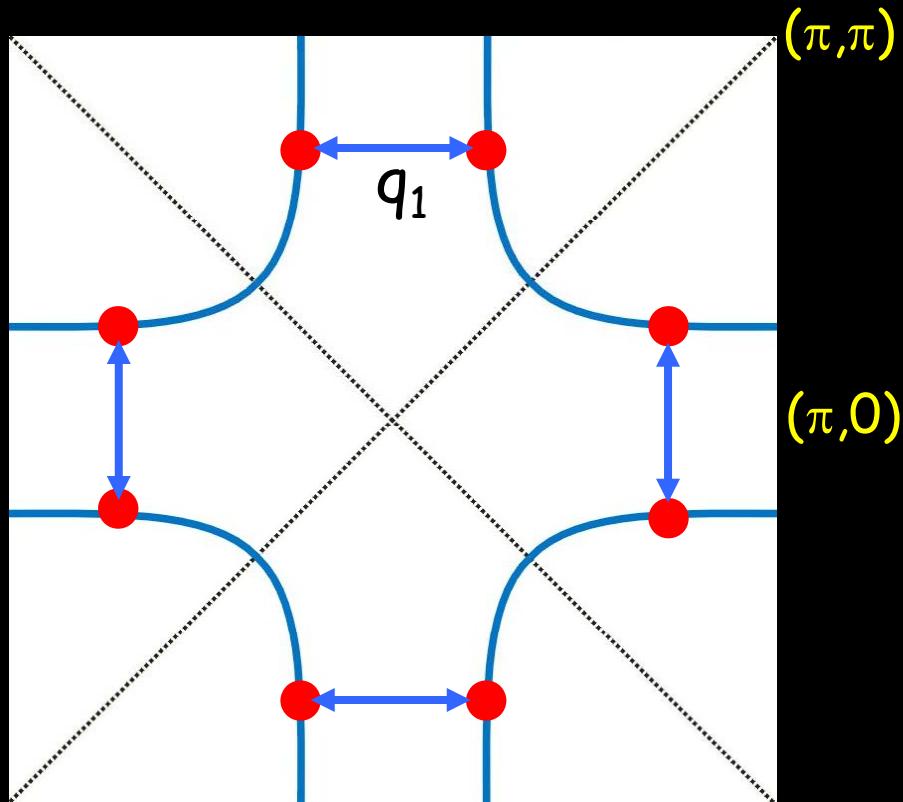


Dispersion:

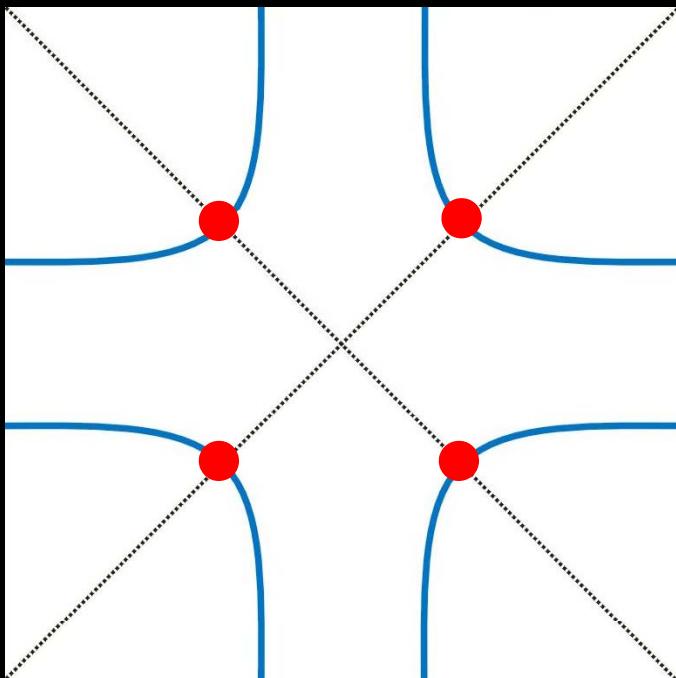
how does each  $\vec{q}_i$  vary with  $E$  ?

For example, look at the dispersion of:

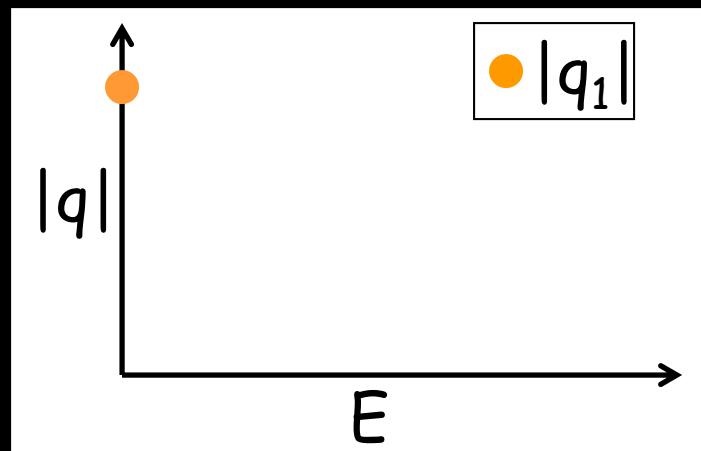
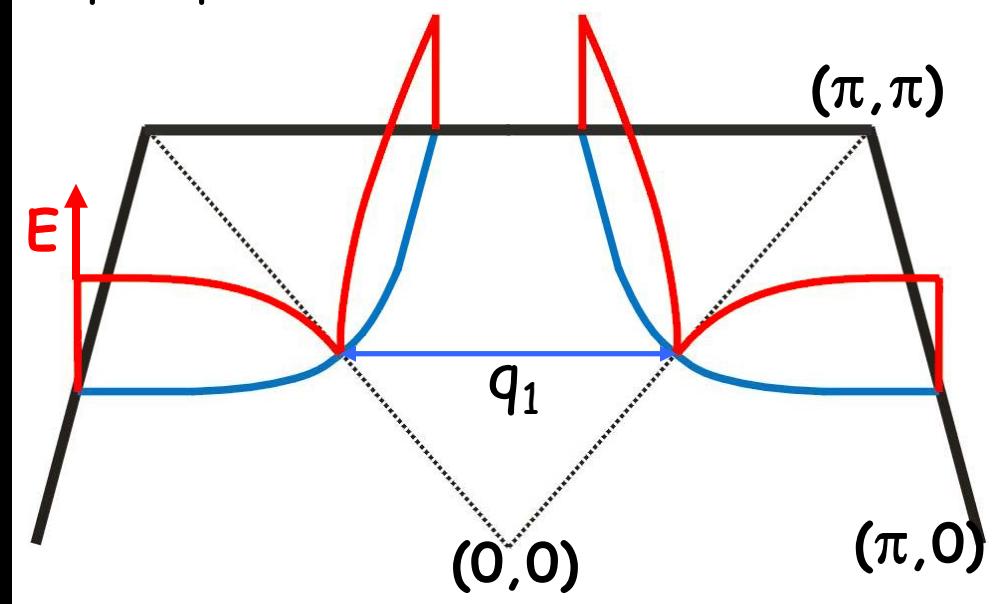
$$\vec{q}_1 \parallel (\pm\pi, 0) \text{ or } (0, \pm\pi)$$



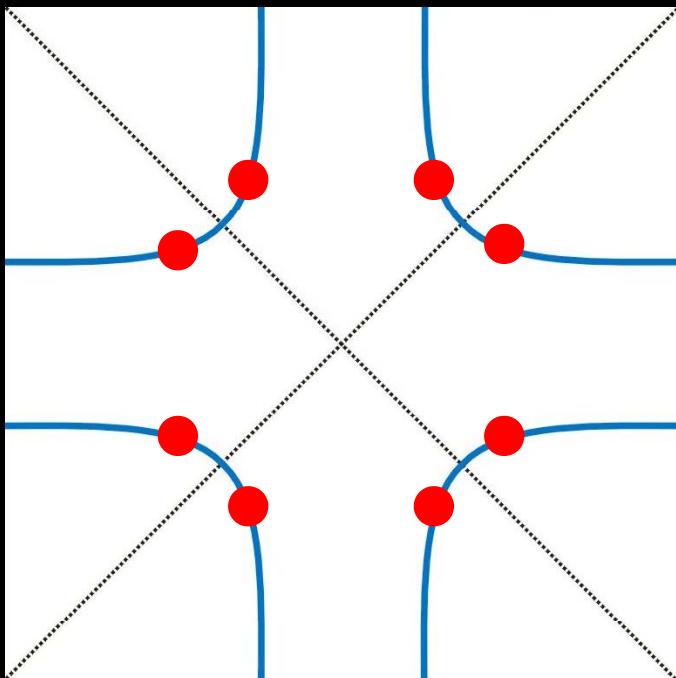
# Expected energy dependence of $|\vec{q}_1|$



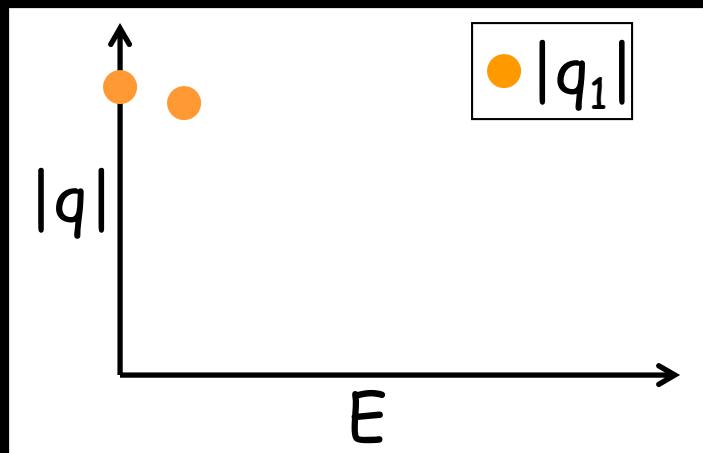
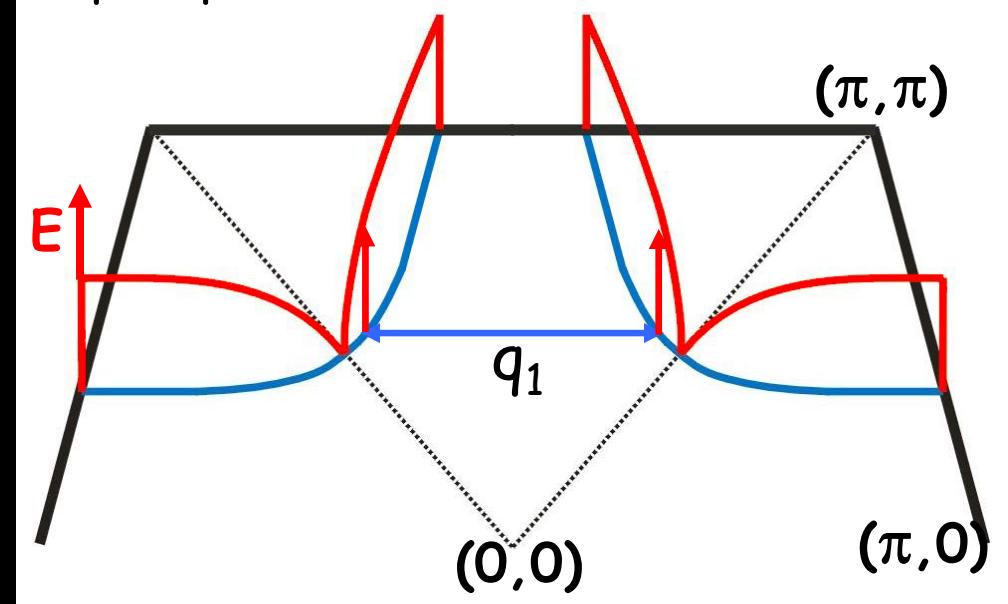
A perspective view:



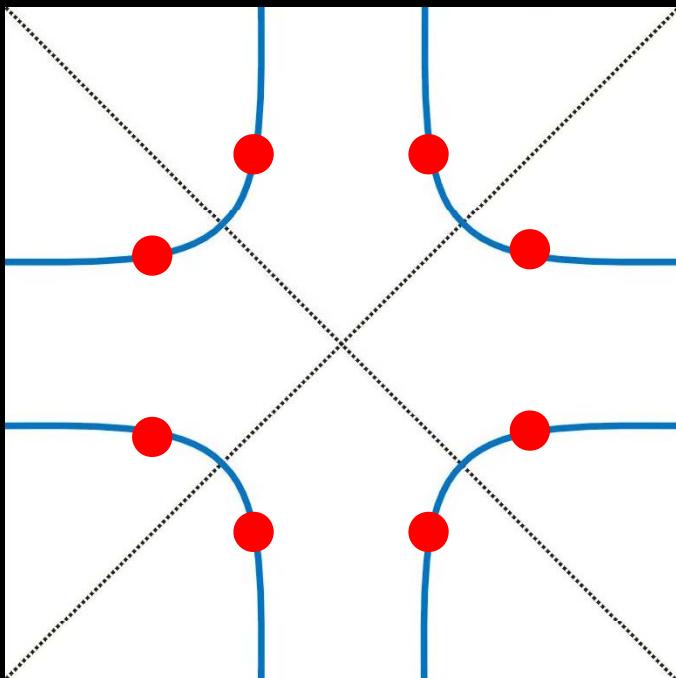
# Expected energy dependence of $|\vec{q}_1|$



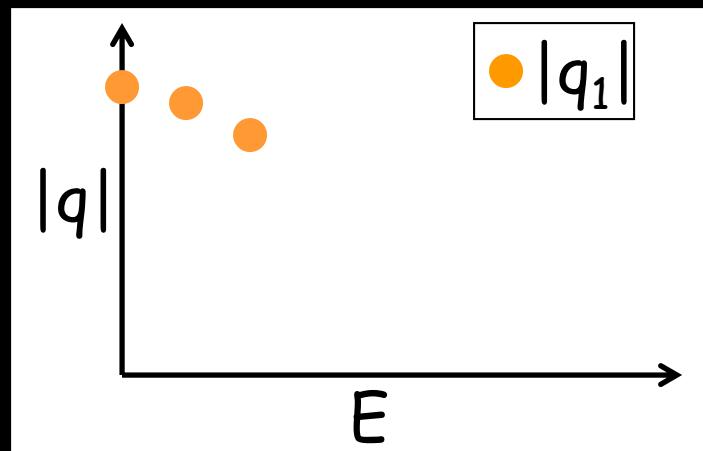
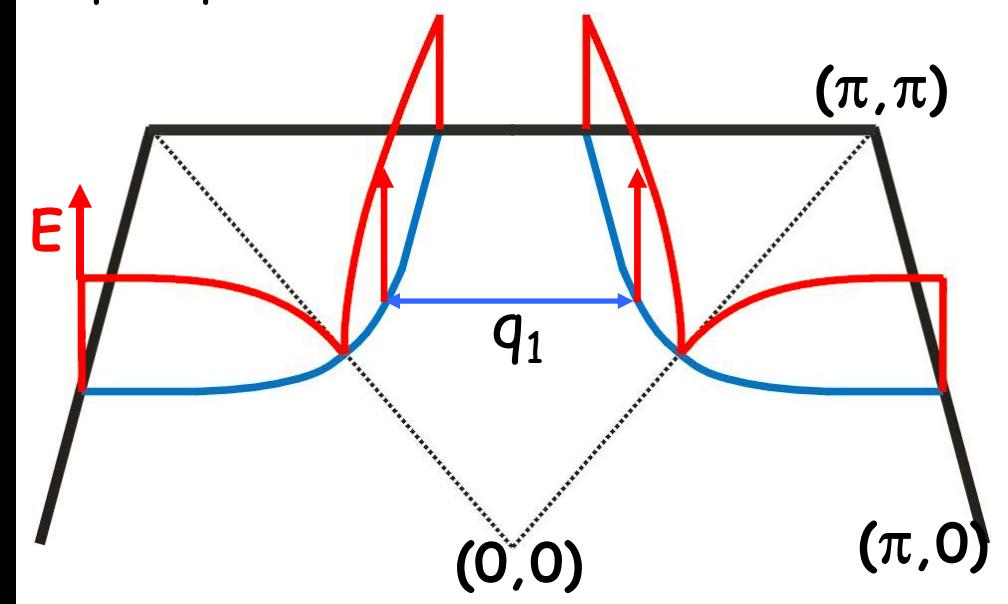
A perspective view:



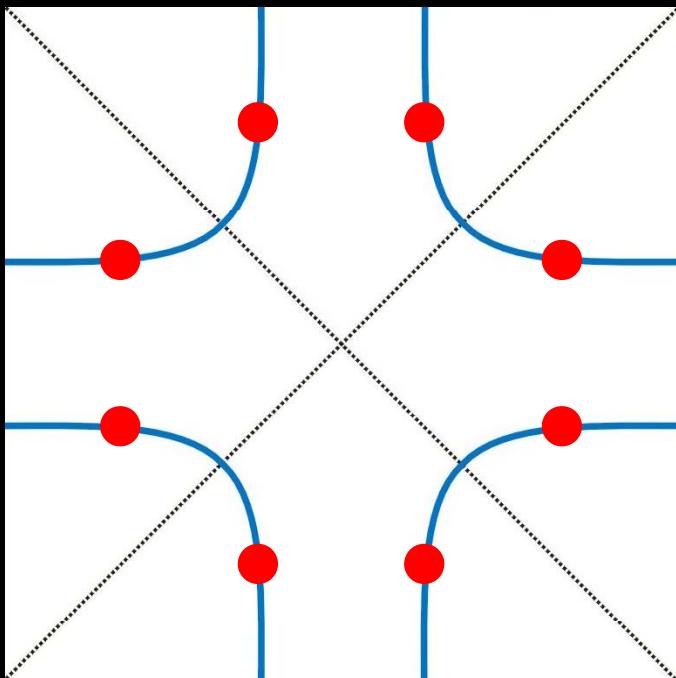
# Expected energy dependence of $|\vec{q}_1|$



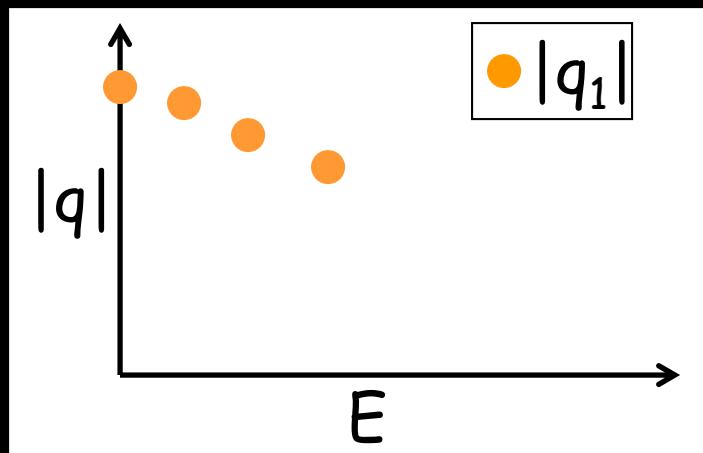
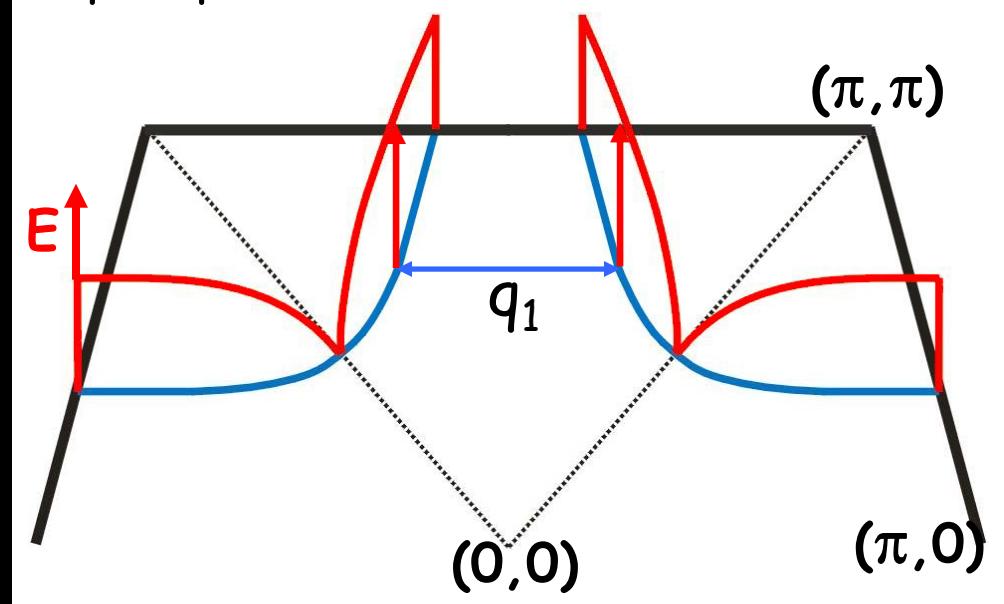
A perspective view:



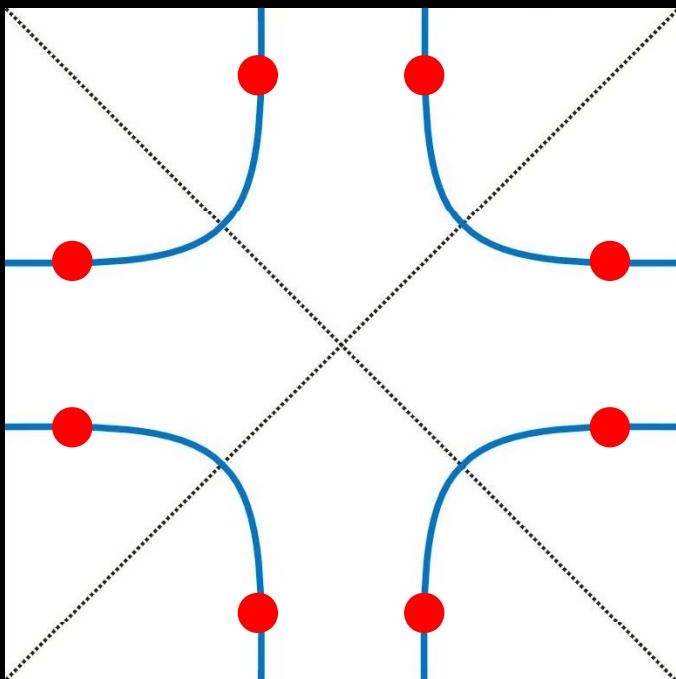
# Expected energy dependence of $|\vec{q}_1|$



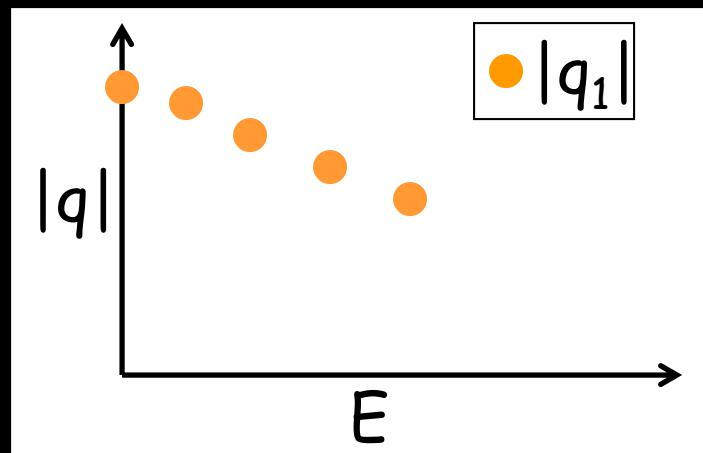
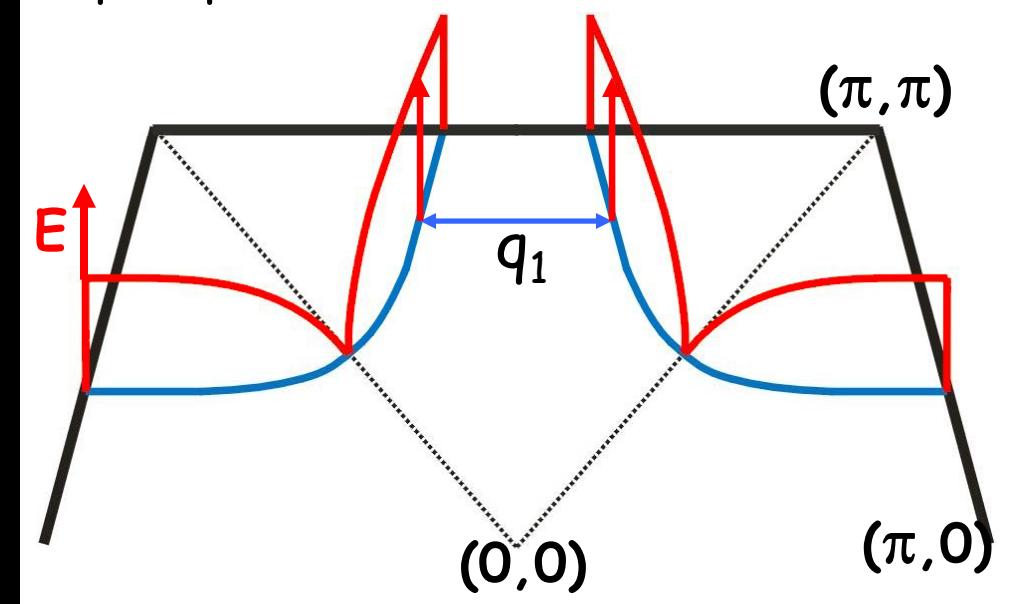
A perspective view:



# Expected energy dependence of $|\vec{q}_1|$

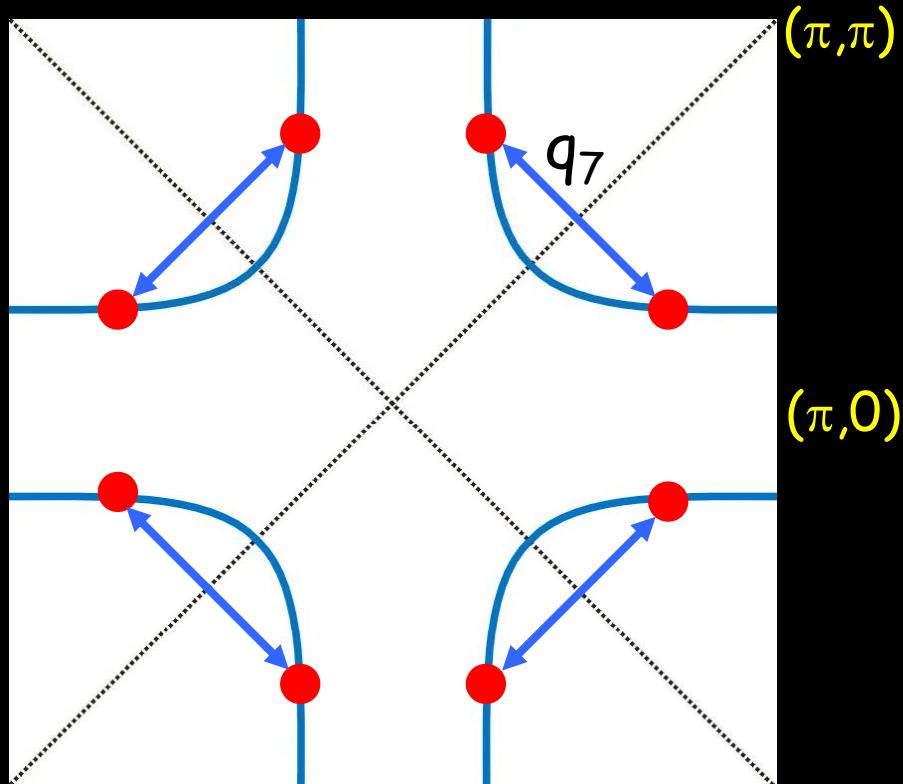


A perspective view:

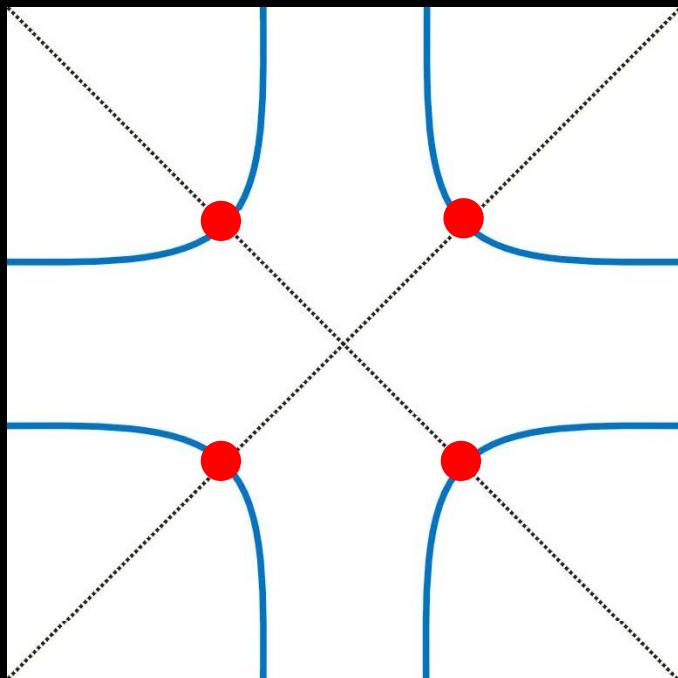


For example, look at the dispersion of:

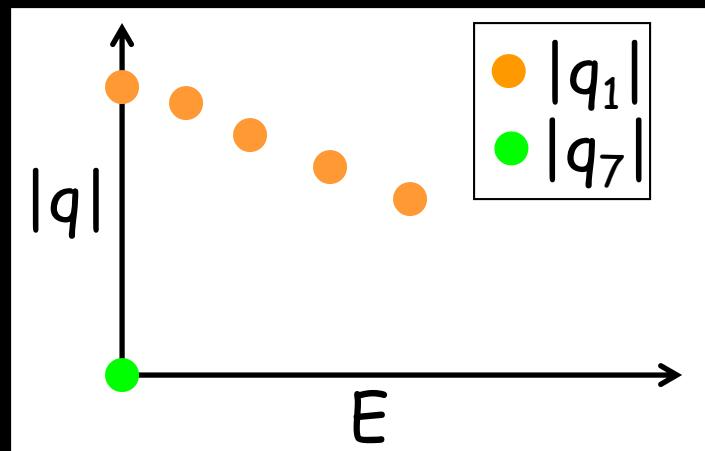
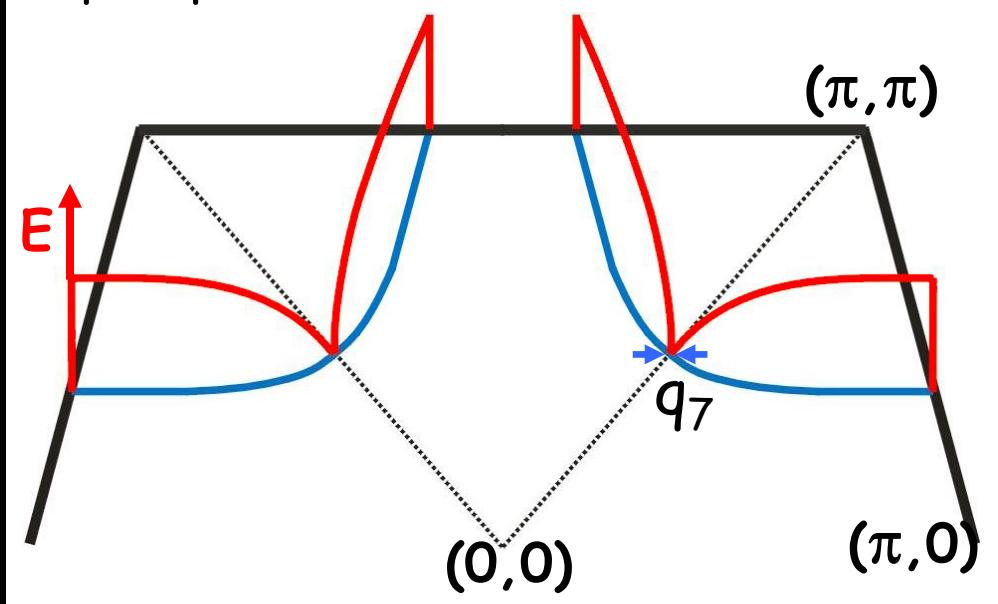
$$\vec{q}_7 \parallel (\pm\pi, \pm\pi)$$



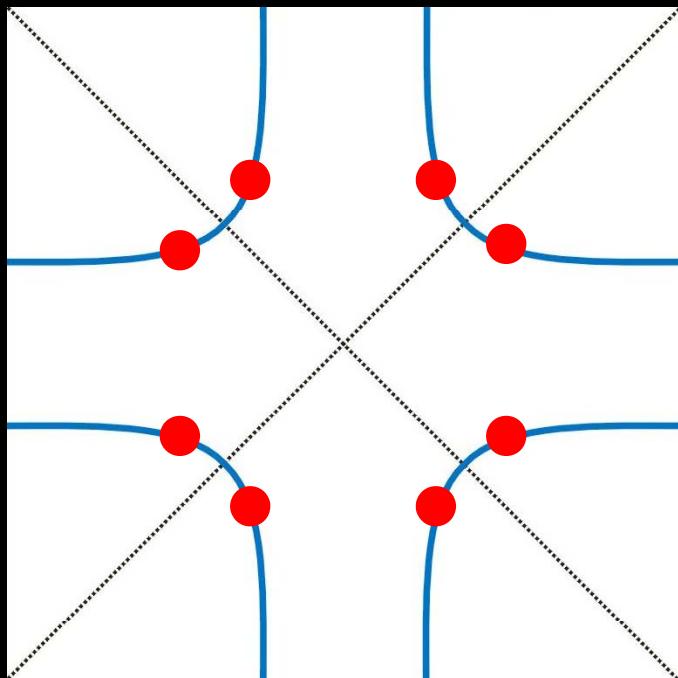
# Expected energy dependence of $|\vec{q}_7|$



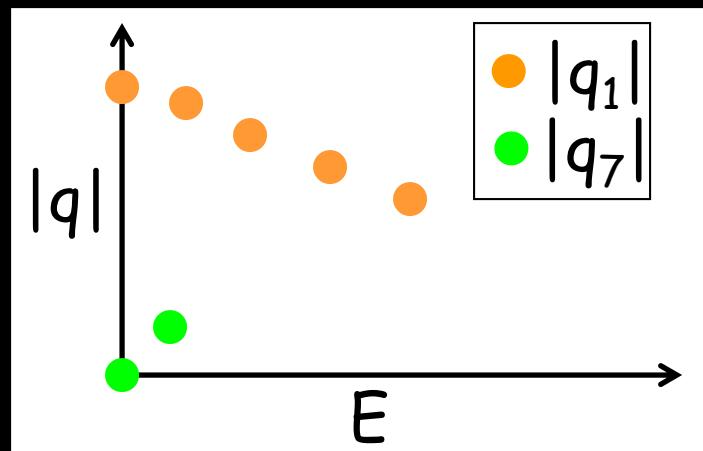
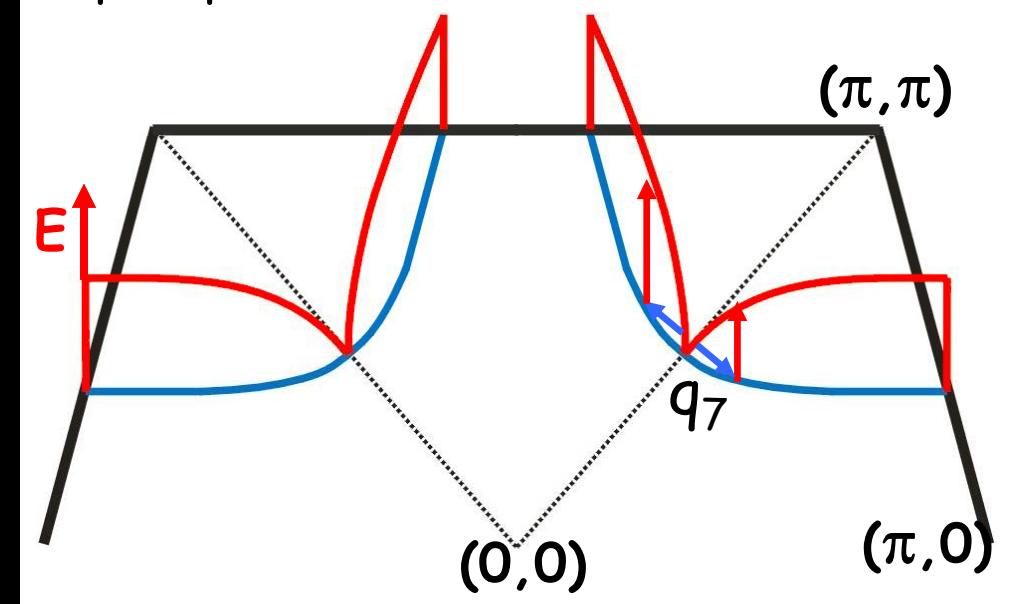
A perspective view:



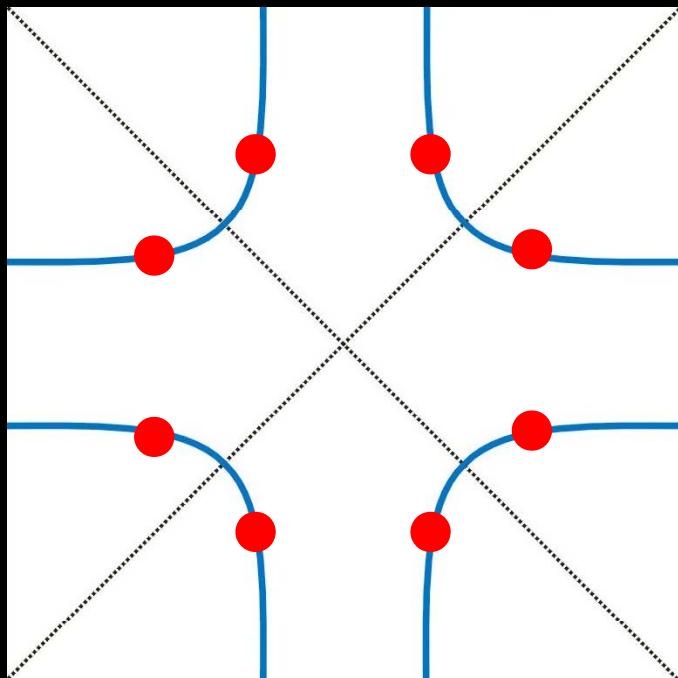
# Expected energy dependence of $|\vec{q}_7|$



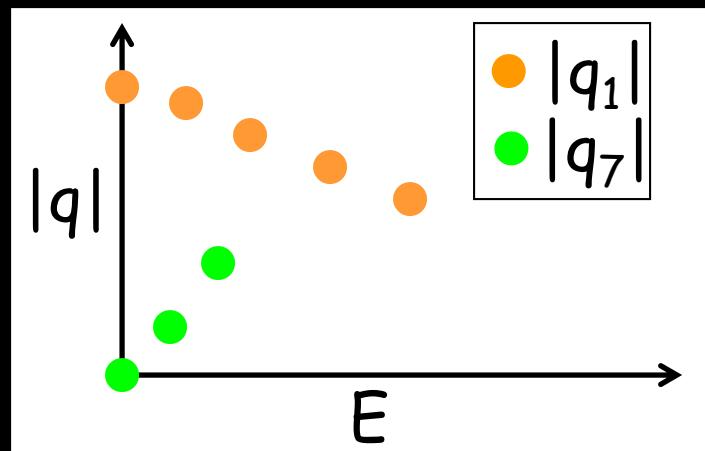
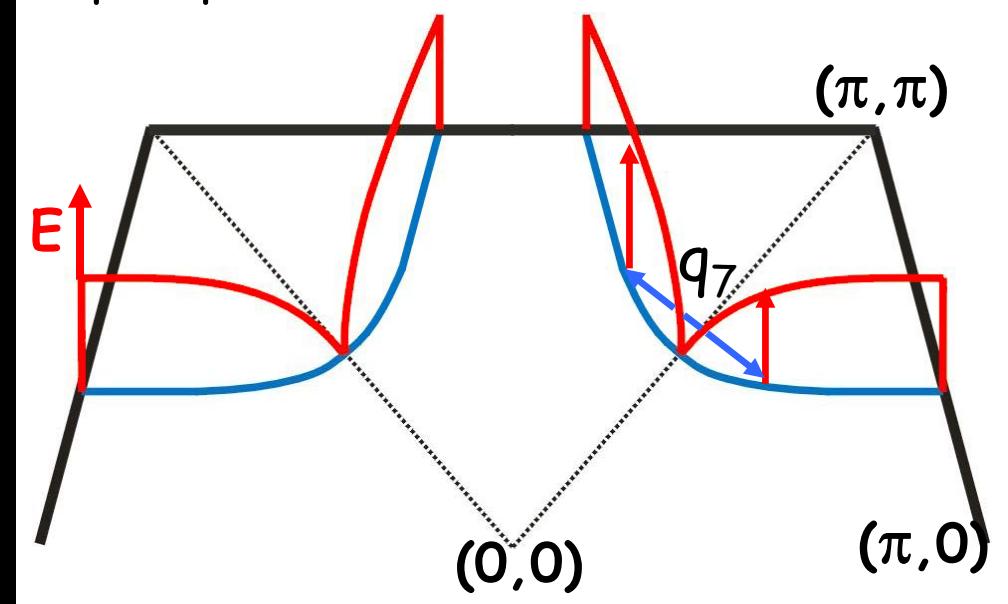
A perspective view:



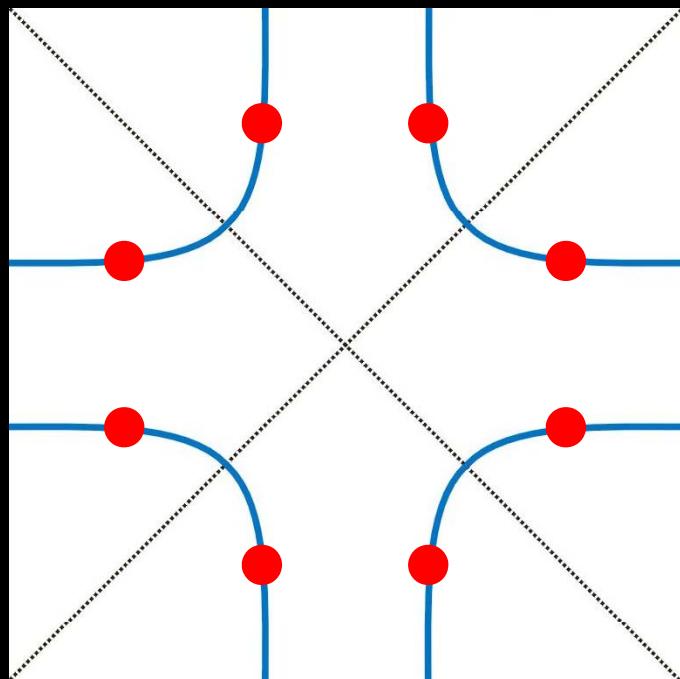
# Expected energy dependence of $|\vec{q}_7|$



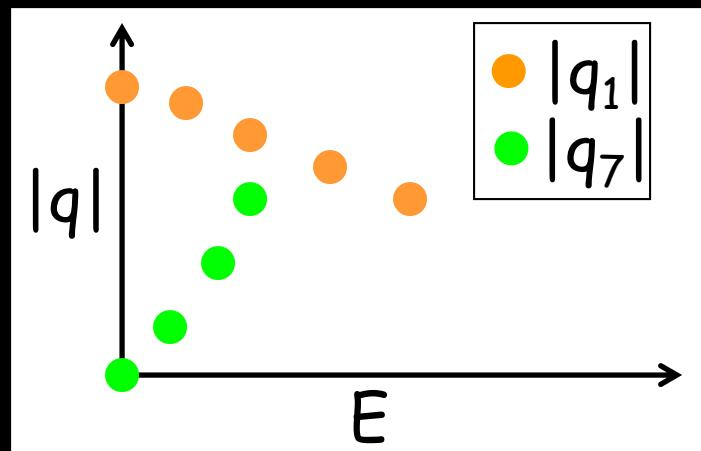
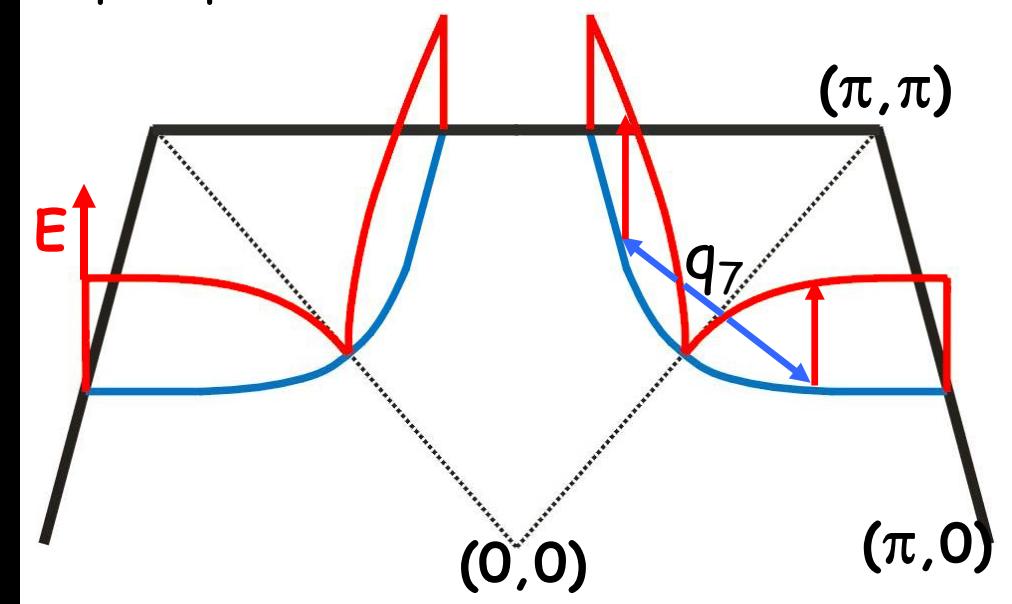
A perspective view:



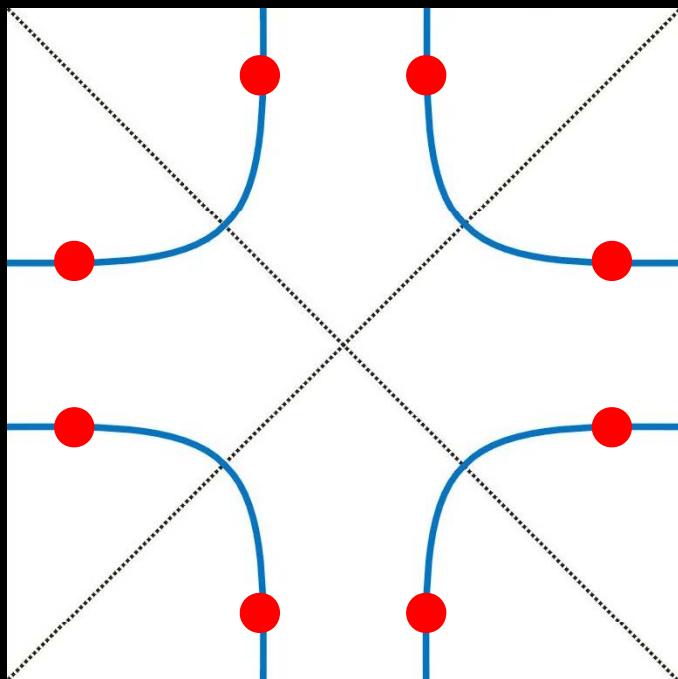
# Expected energy dependence of $|\vec{q}_7|$



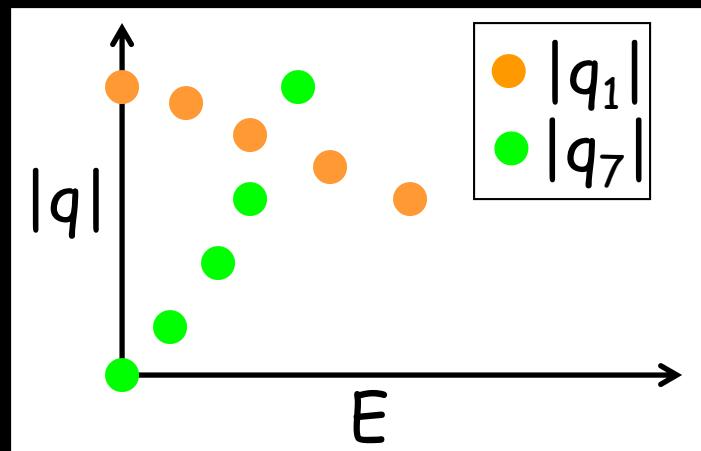
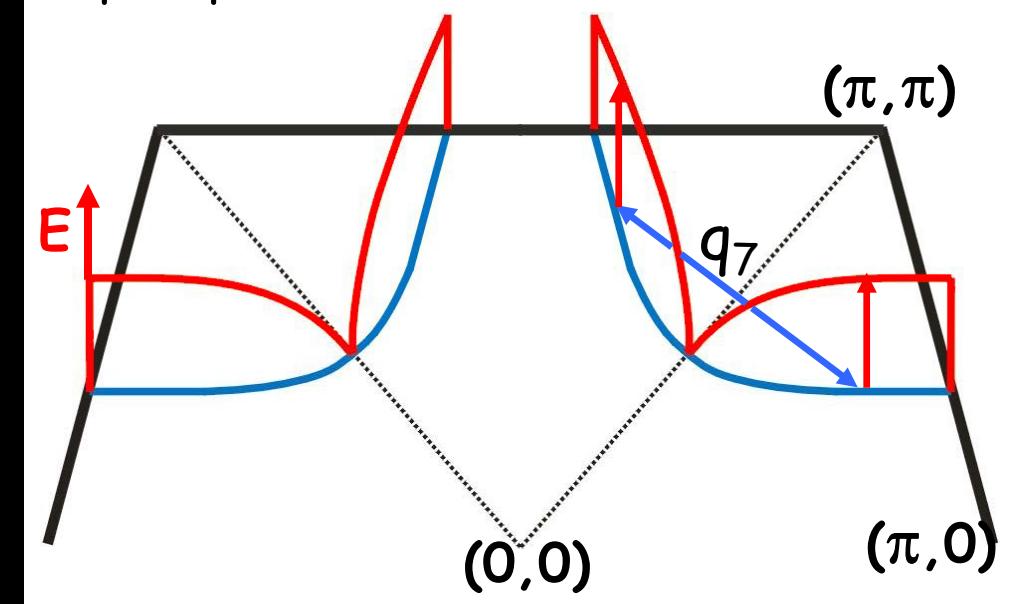
A perspective view:



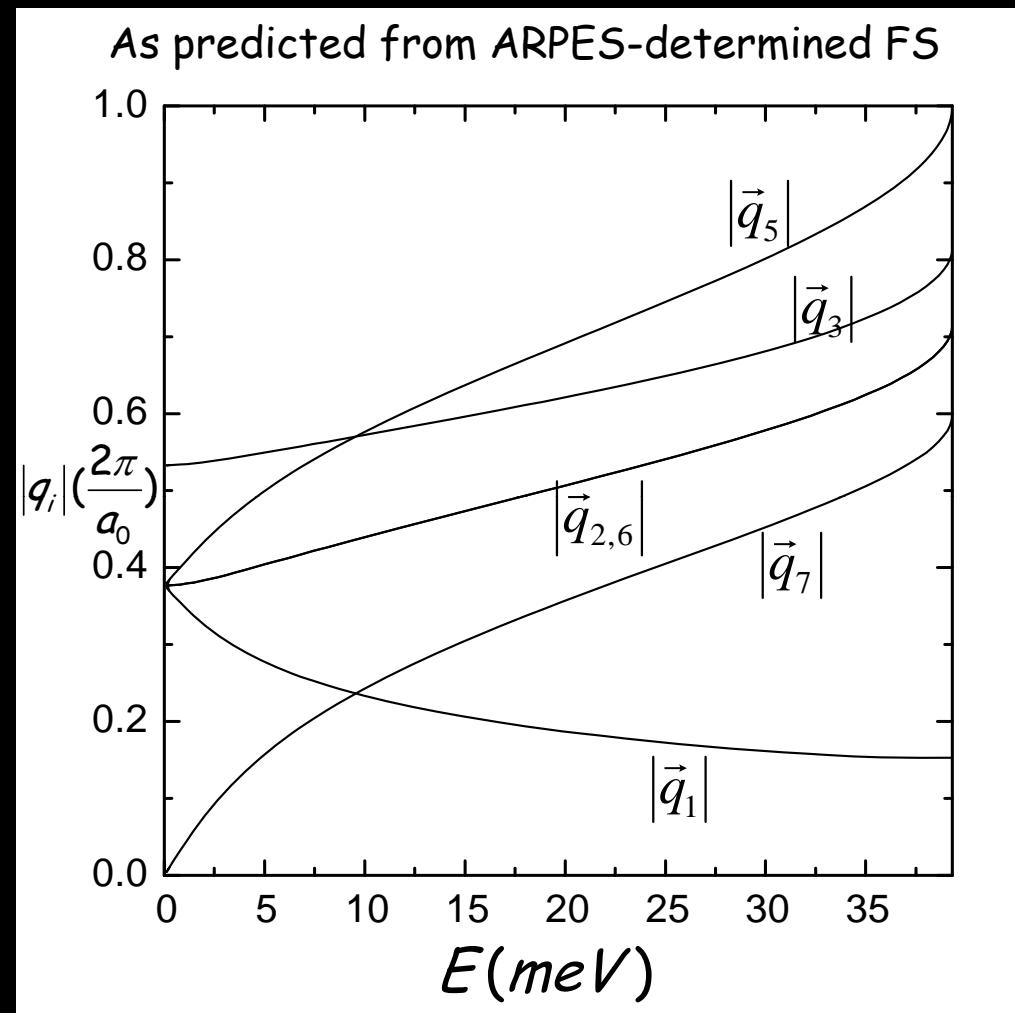
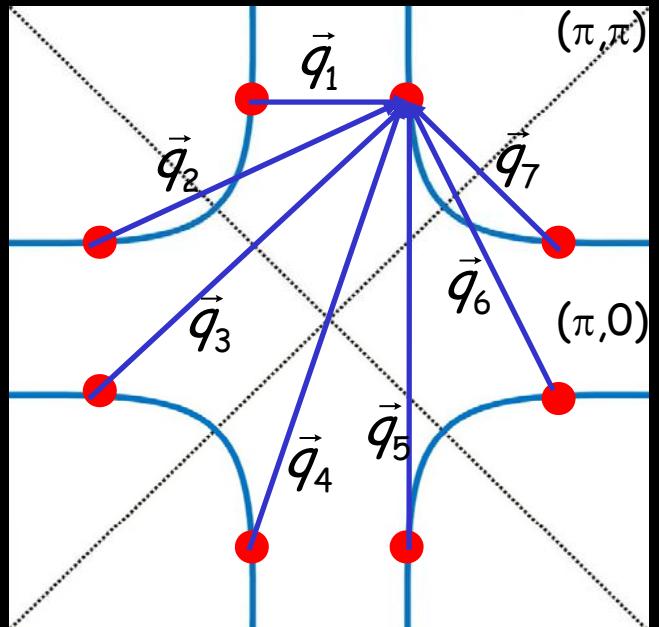
# Expected energy dependence of $|\vec{q}_7|$



A perspective view:

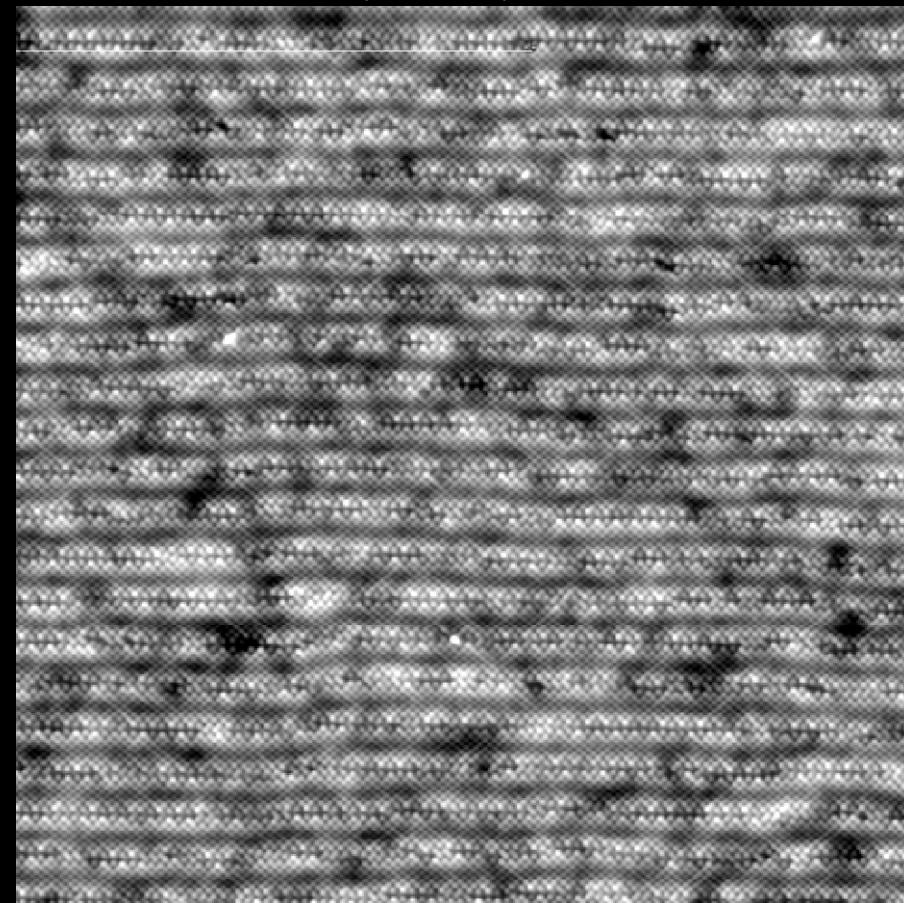


# Expected energy dependence of 5 independent $\mathbf{q}$ 's



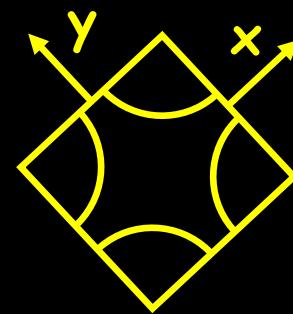
Finally, some data...

topograph

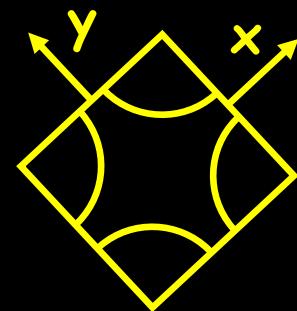
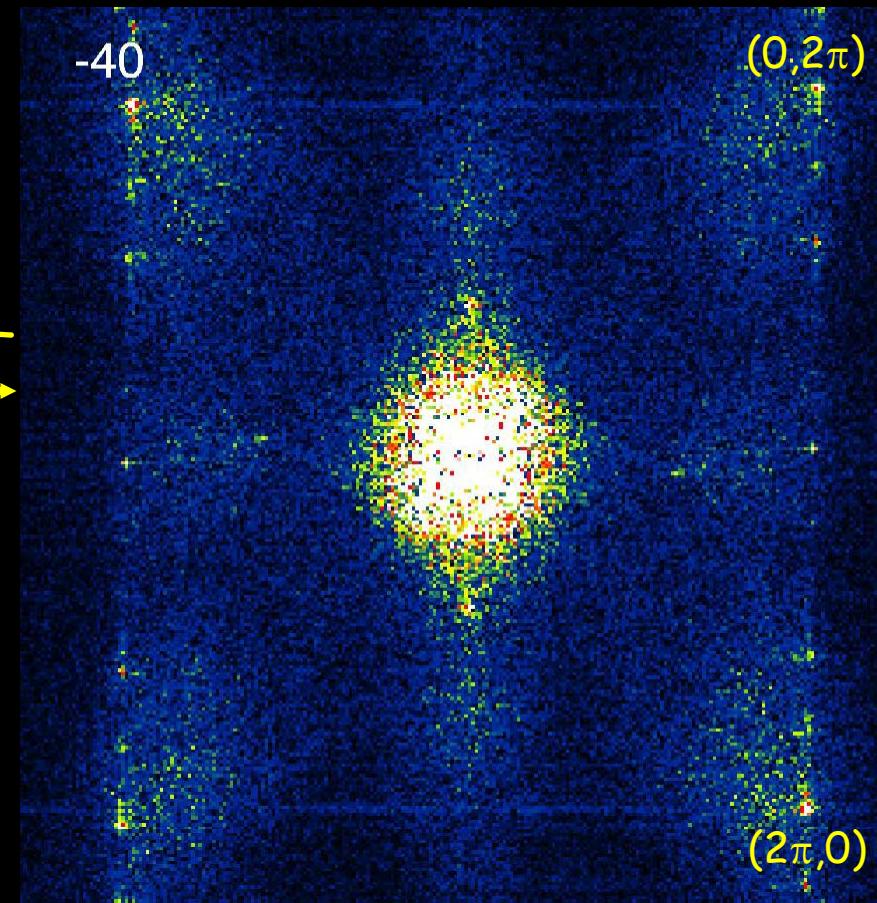
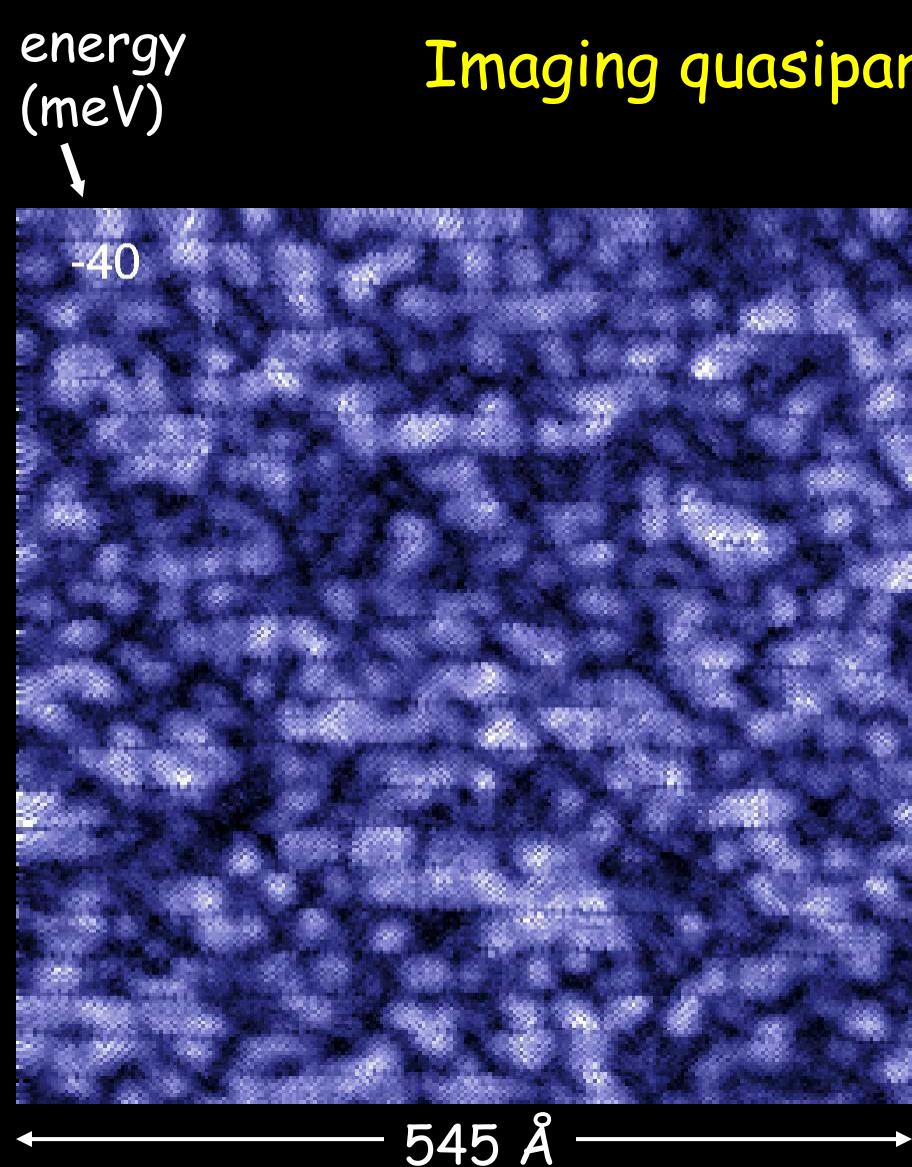


← 545 Å →

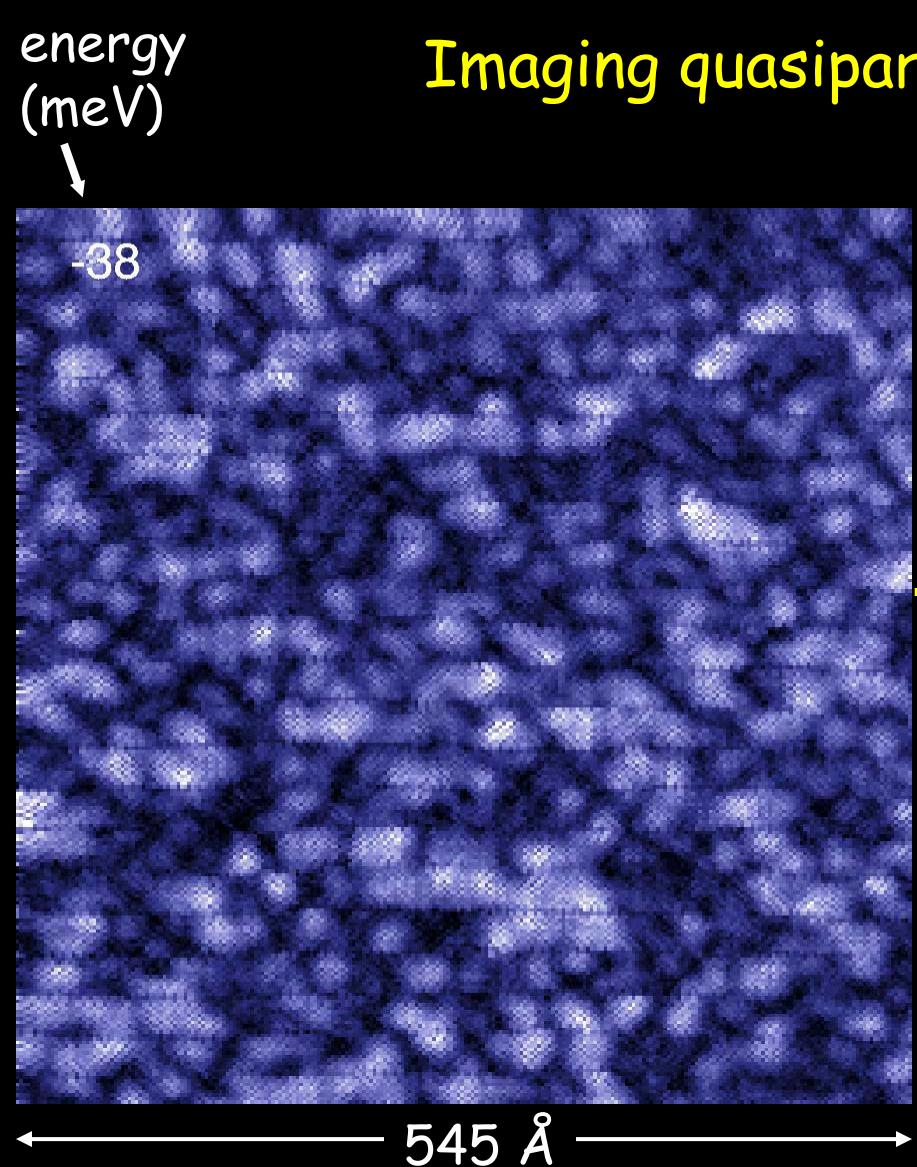
Bi-2212  
 $T_c=76$  K  
 $\Delta \sim 51$  meV



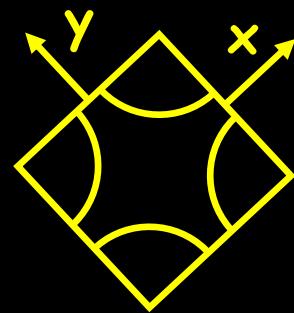
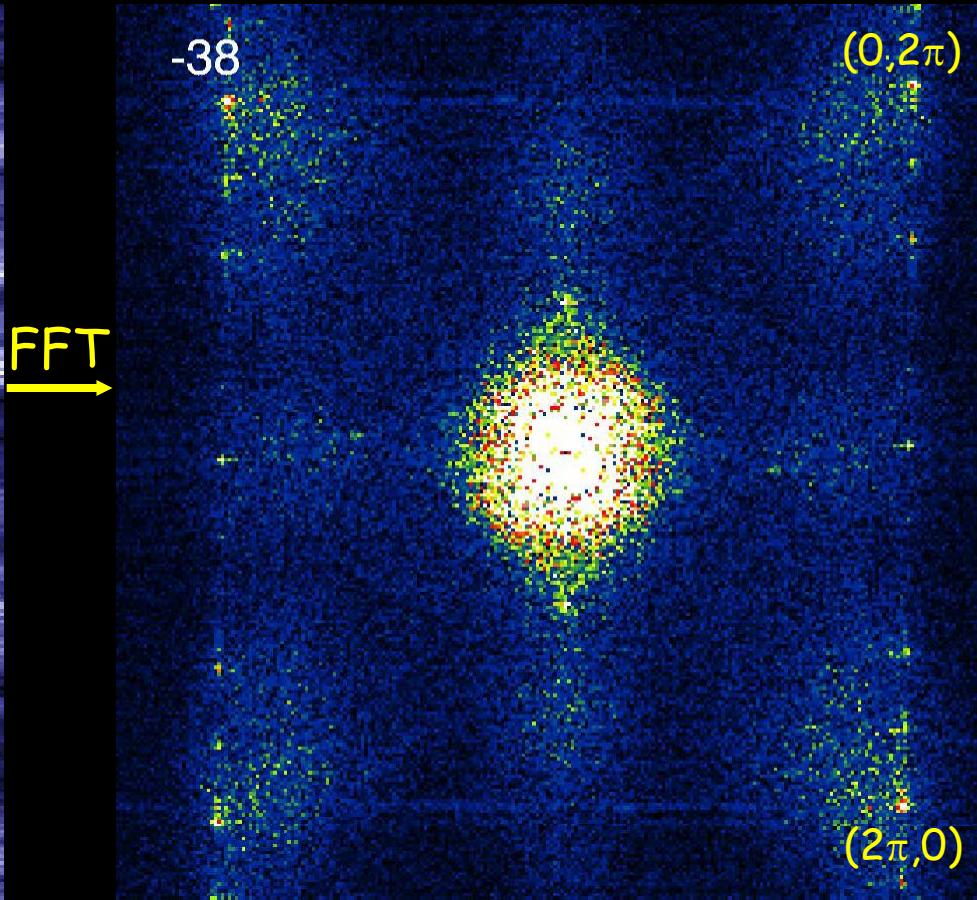
# Imaging quasiparticle wavefunctions



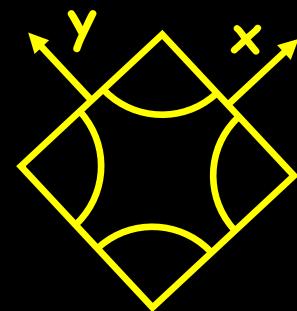
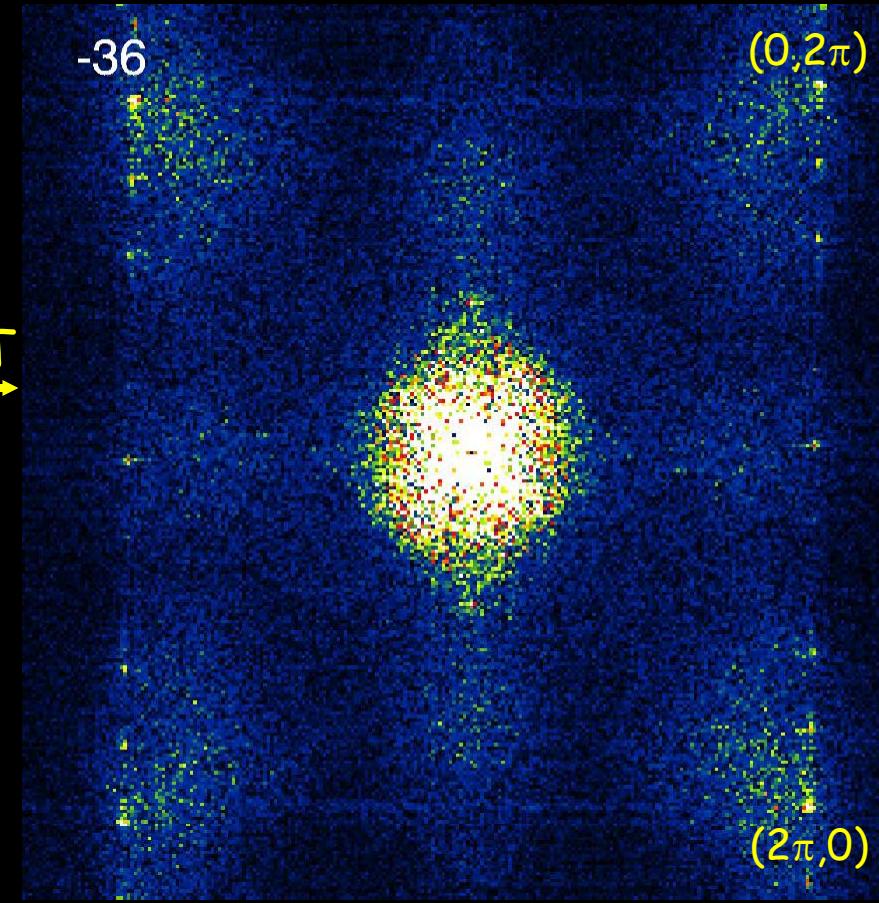
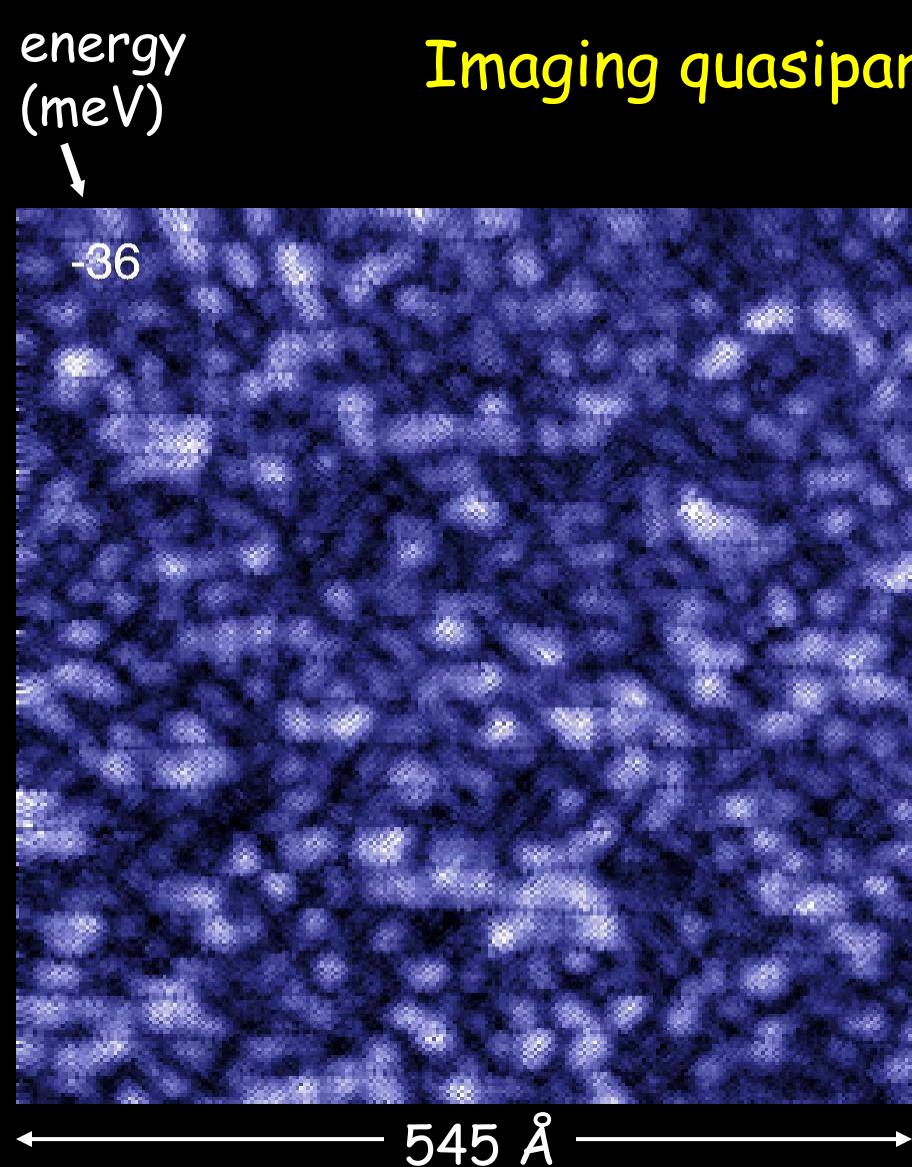
# Imaging quasiparticle wavefunctions

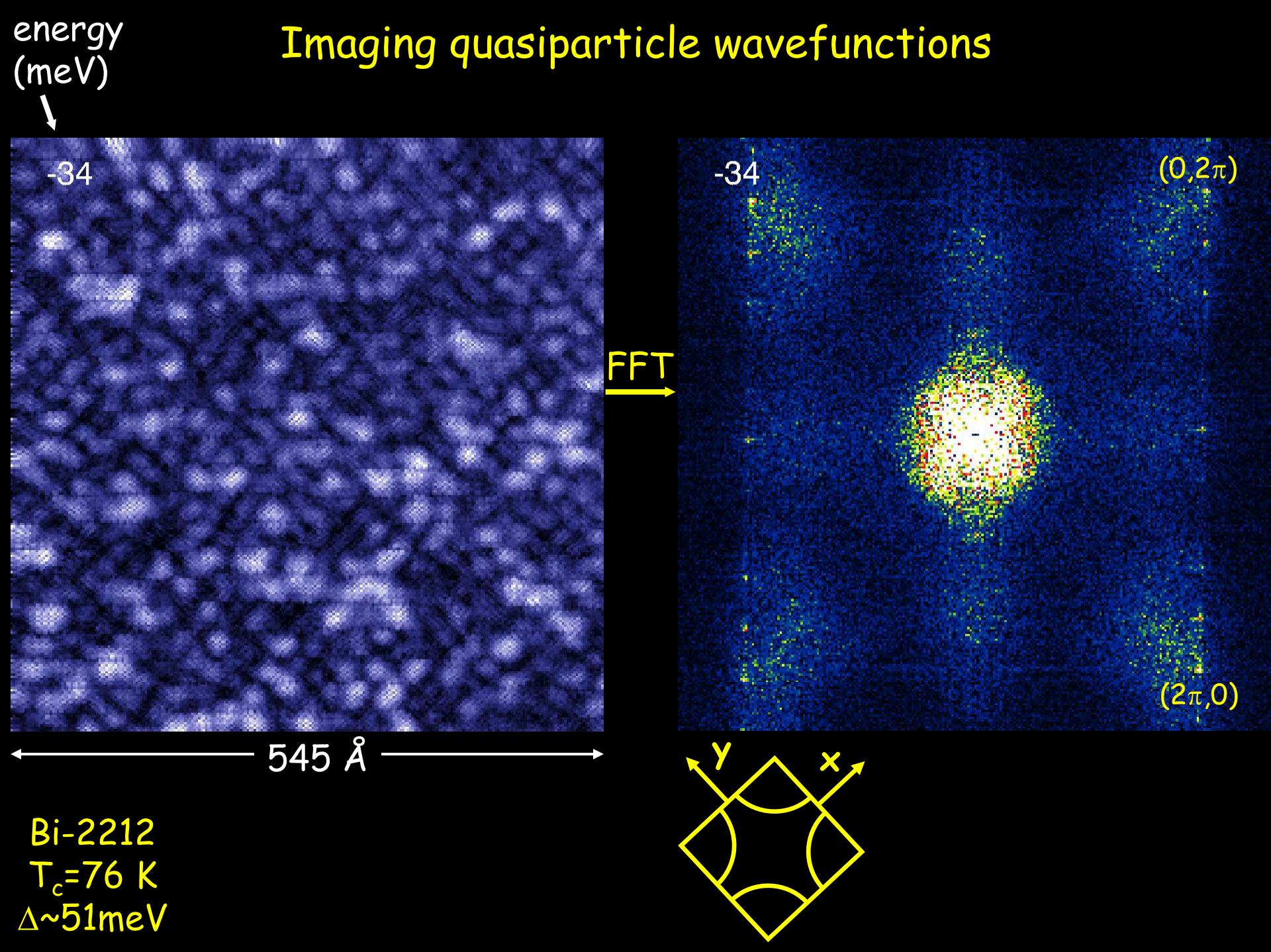


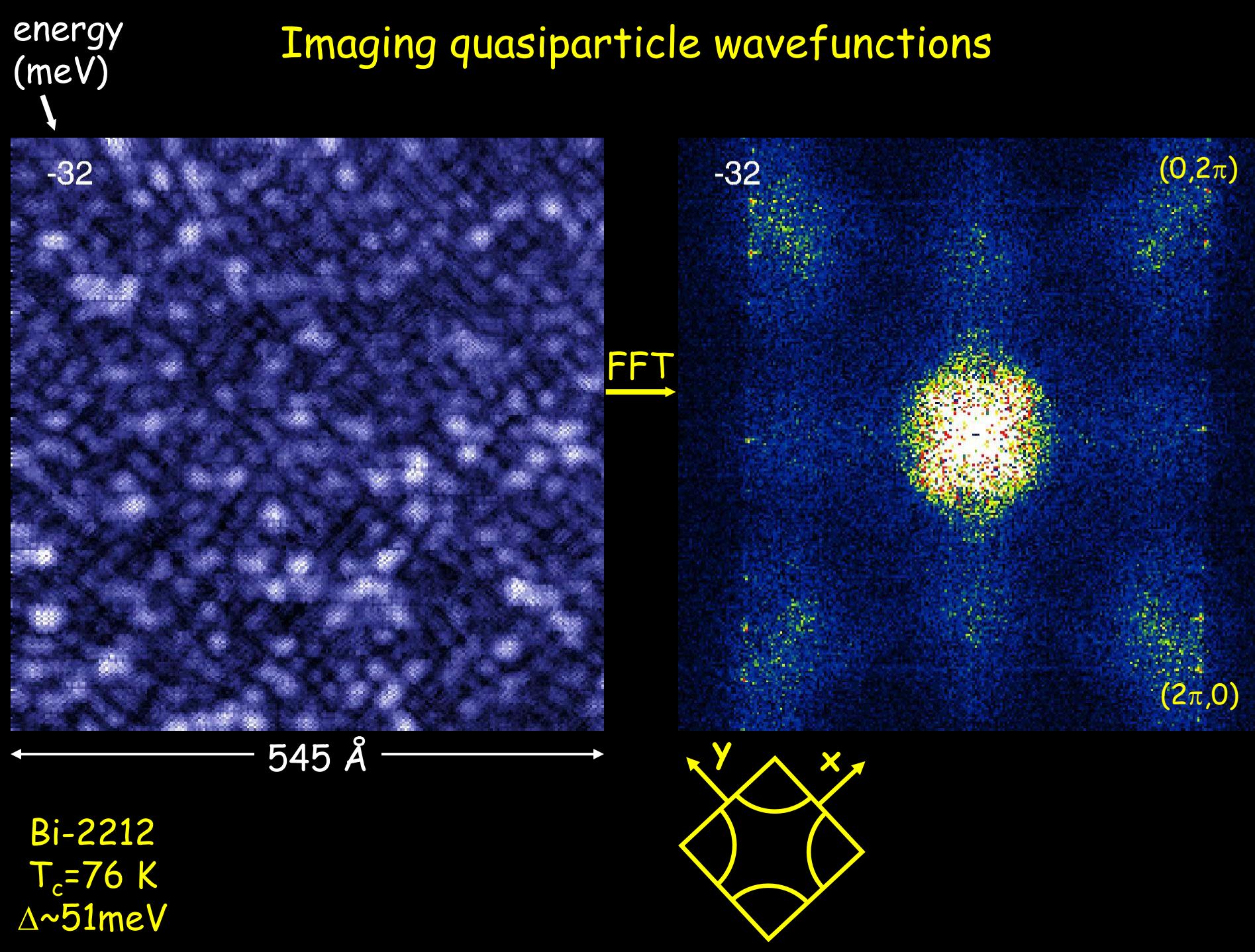
Bi-2212  
 $T_c=76$  K  
 $\Delta \sim 51$  meV



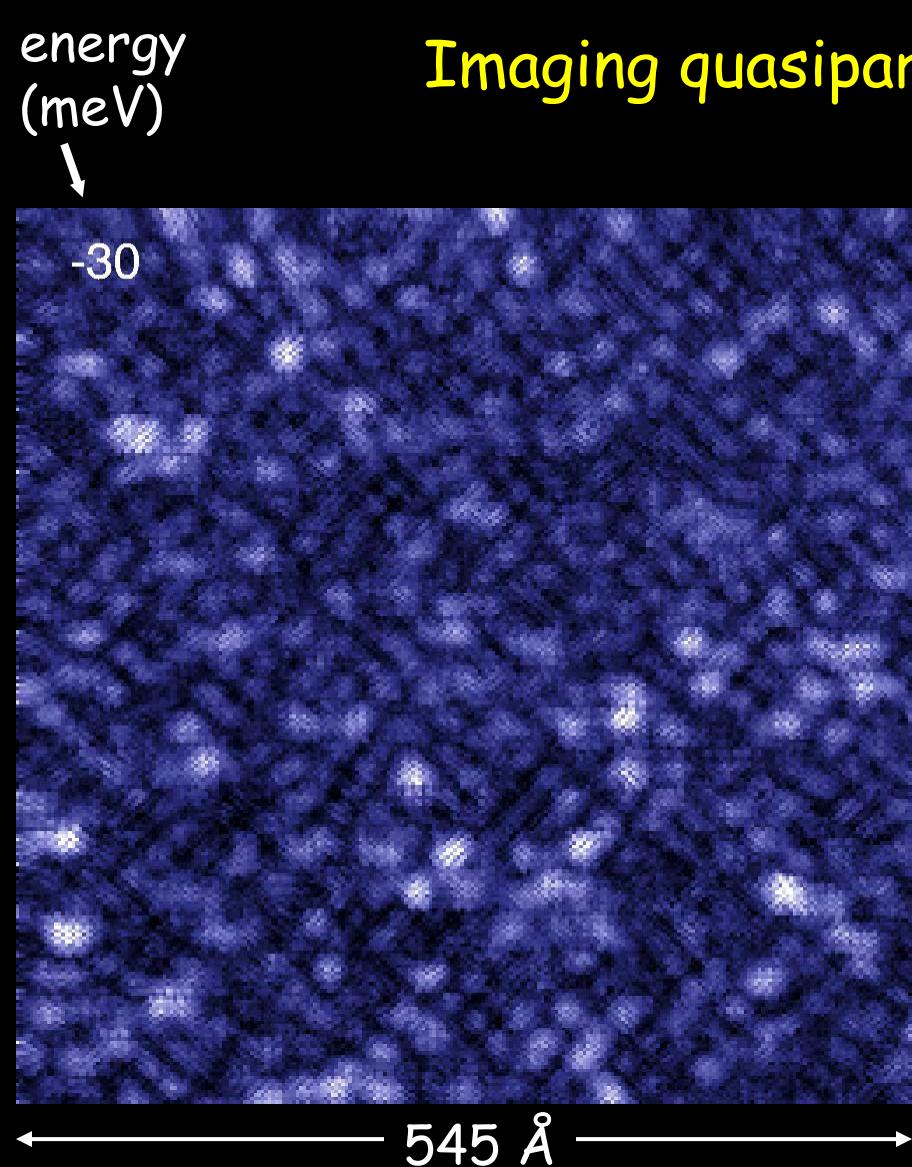
# Imaging quasiparticle wavefunctions



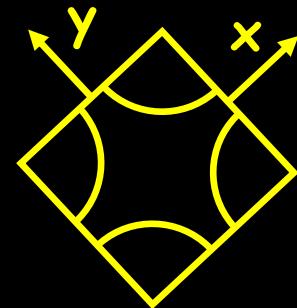
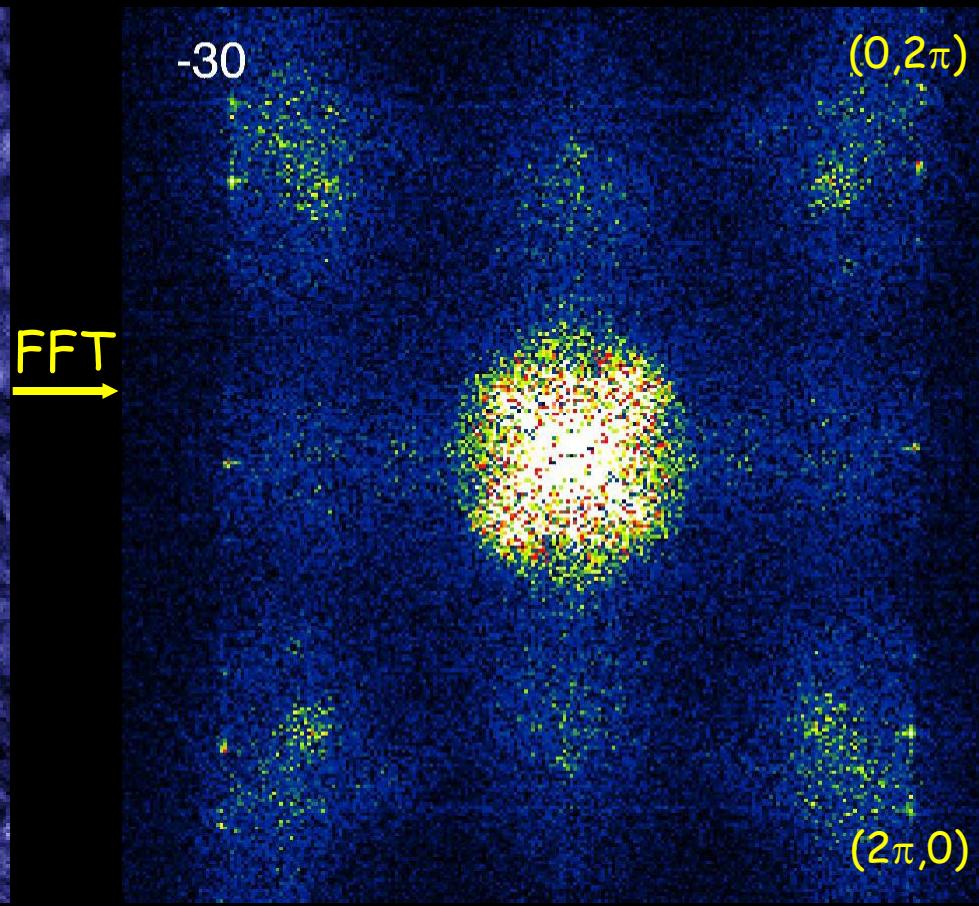


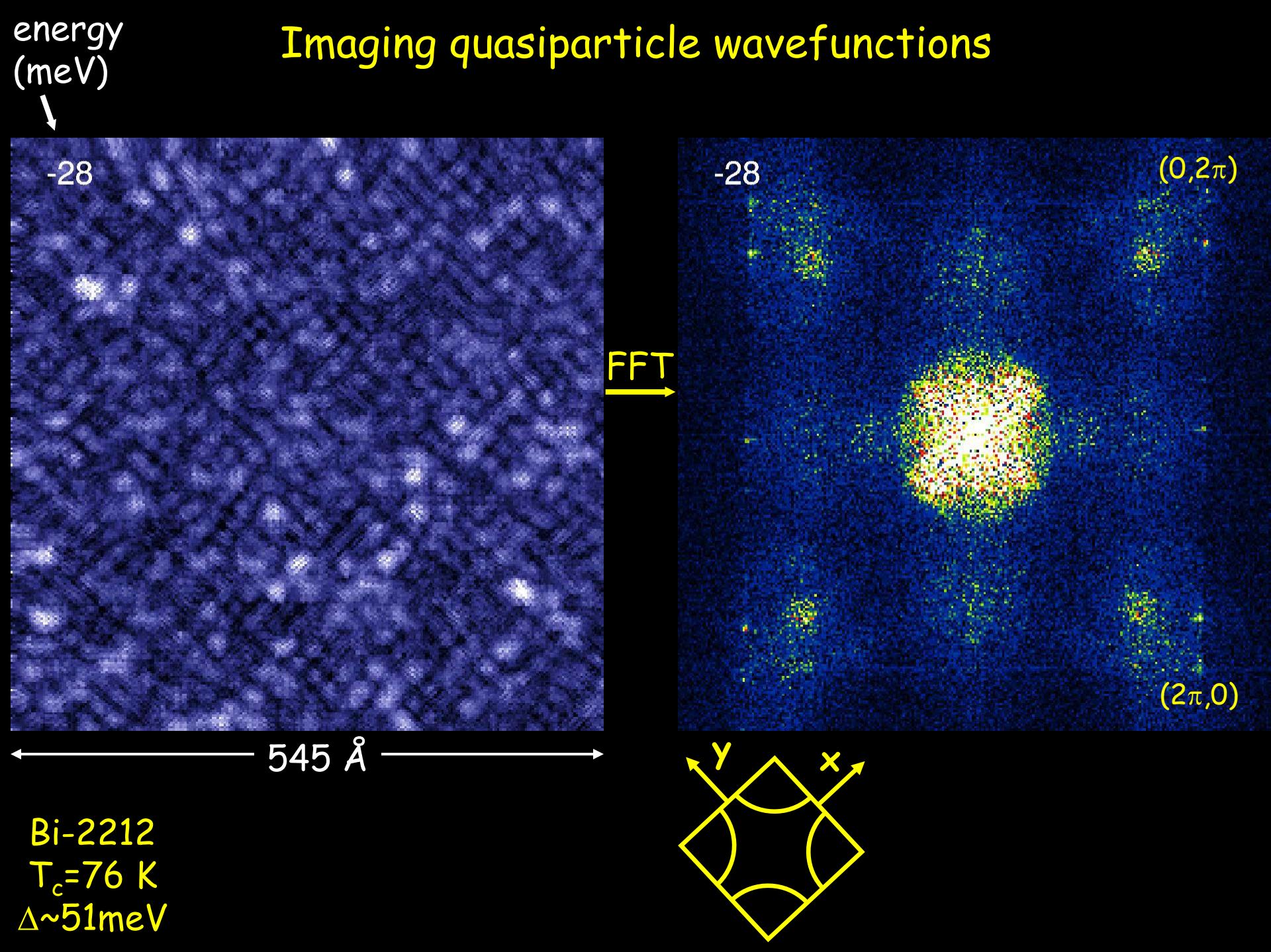


# Imaging quasiparticle wavefunctions

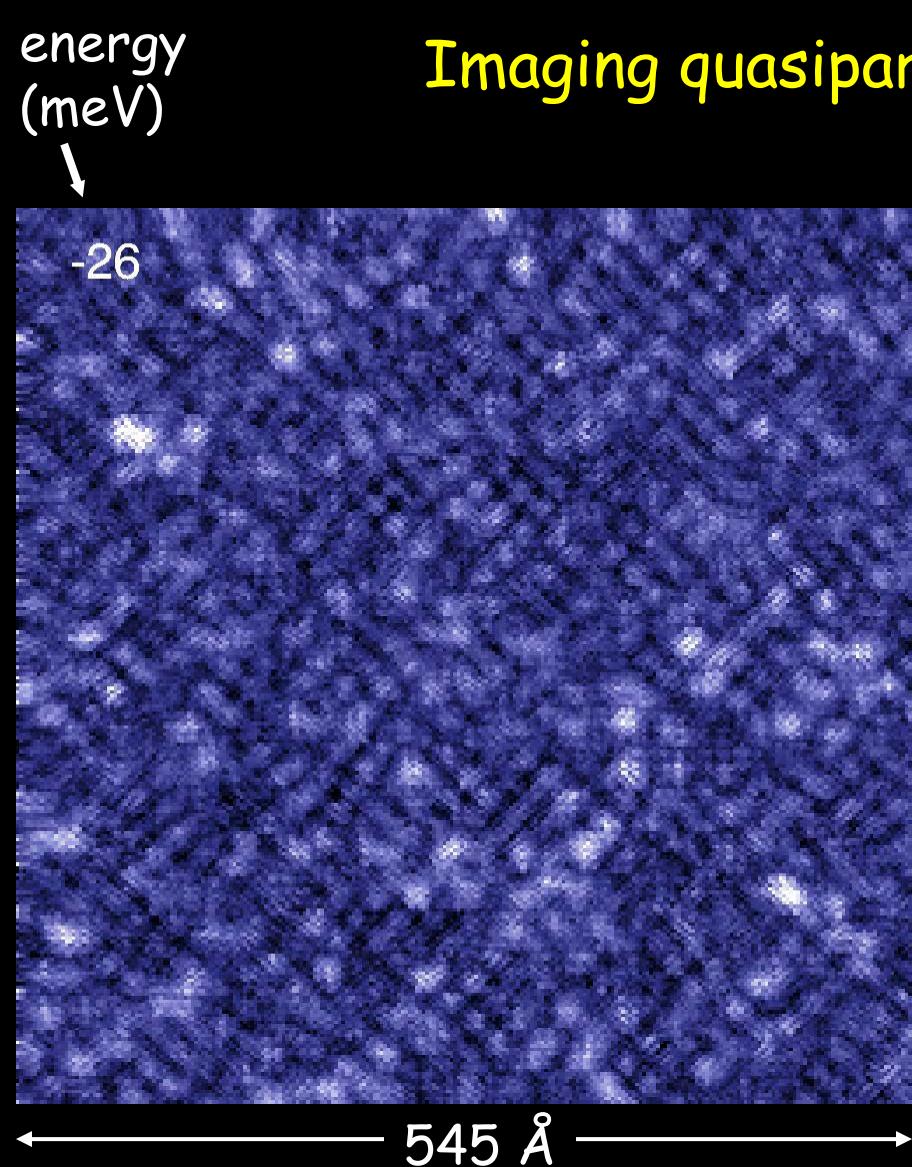


Bi-2212  
 $T_c=76$  K  
 $\Delta \sim 51$  meV

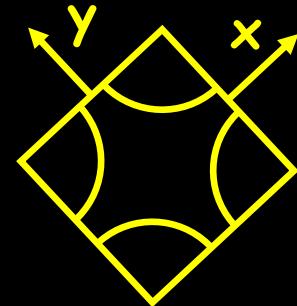
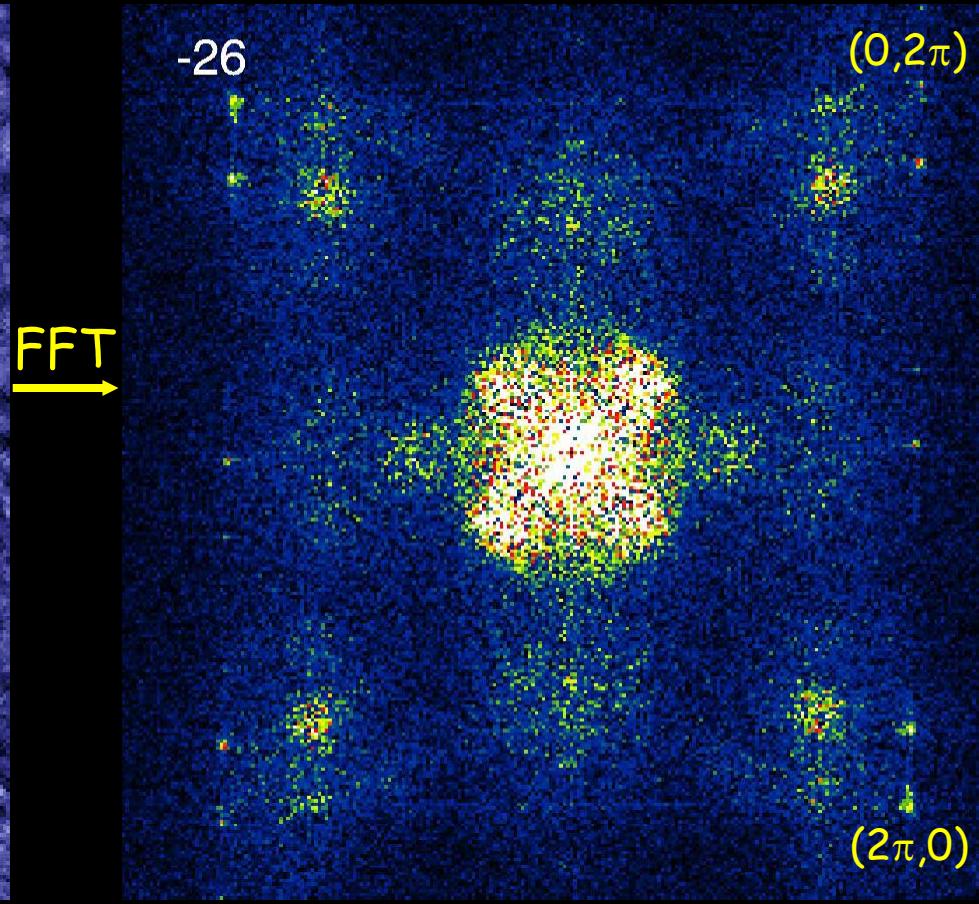




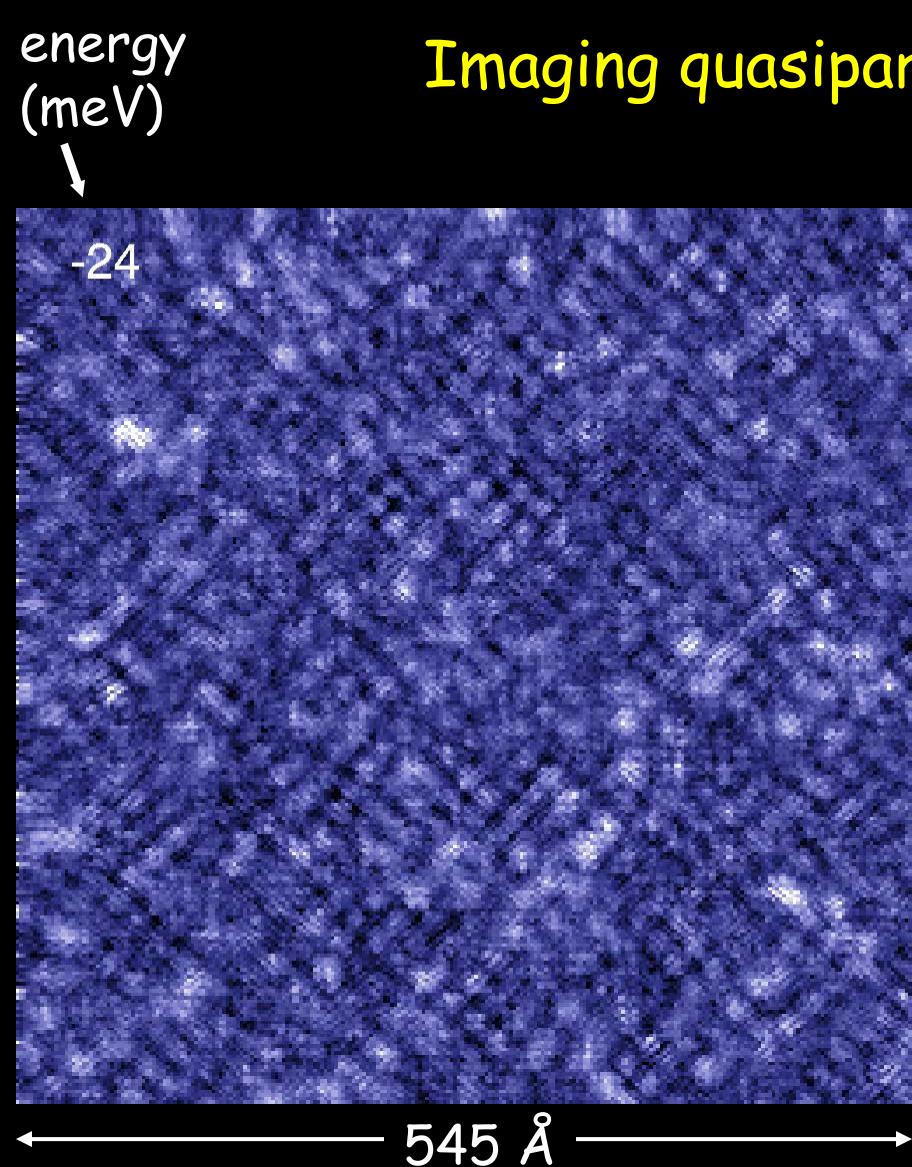
# Imaging quasiparticle wavefunctions



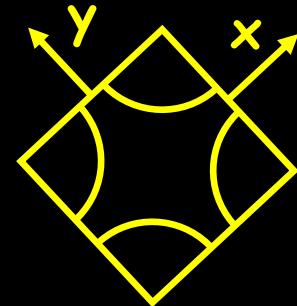
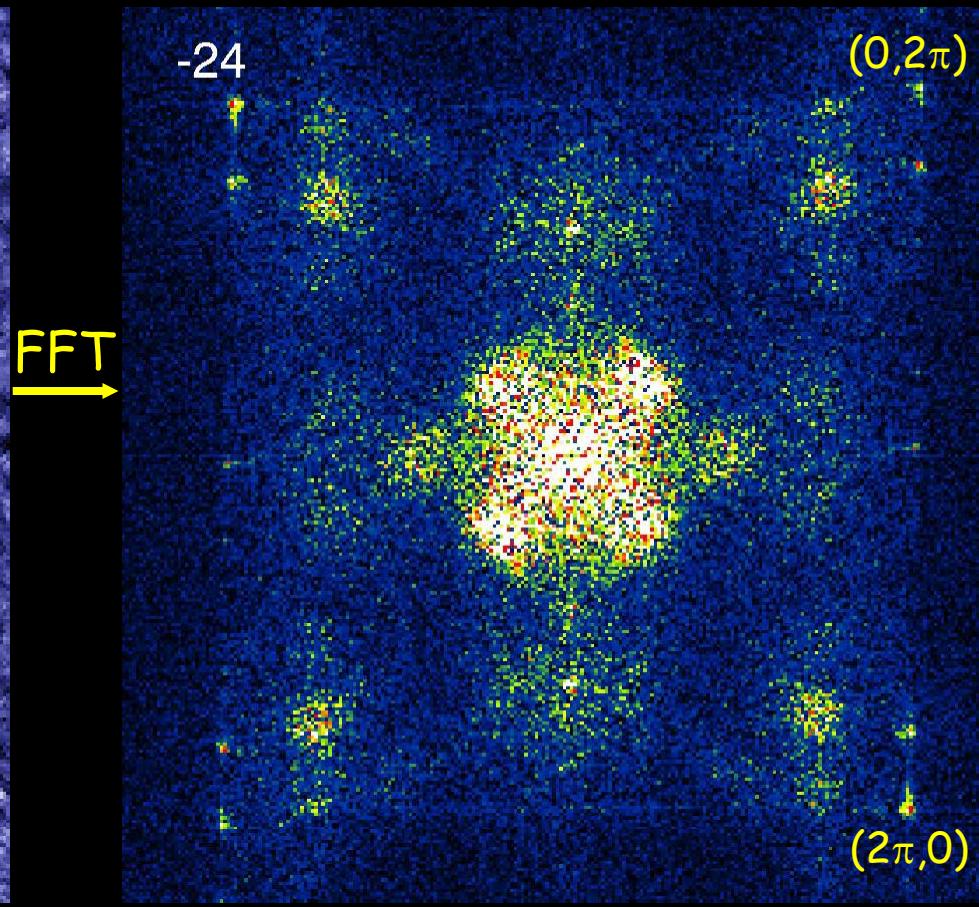
Bi-2212  
 $T_c=76$  K  
 $\Delta \sim 51$  meV

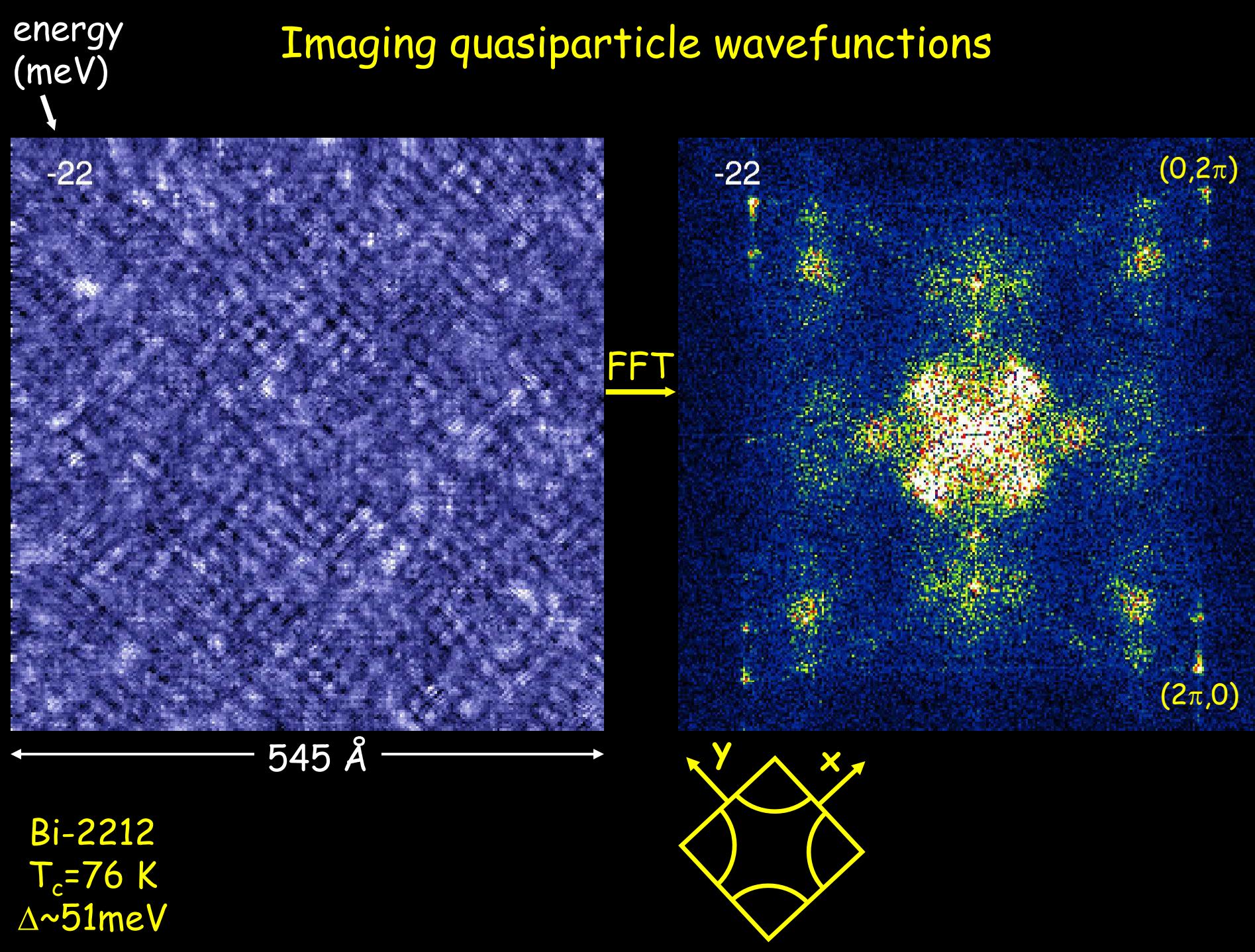


# Imaging quasiparticle wavefunctions

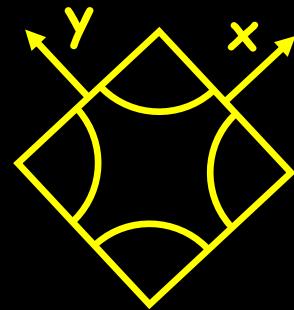
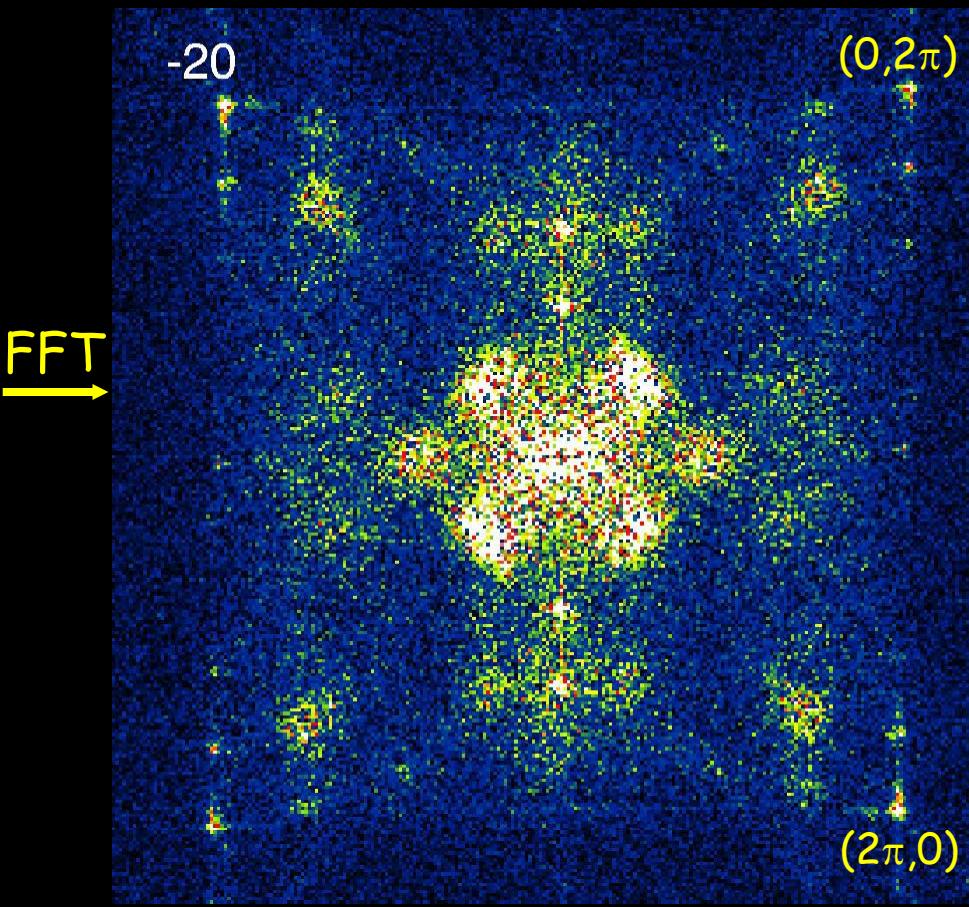
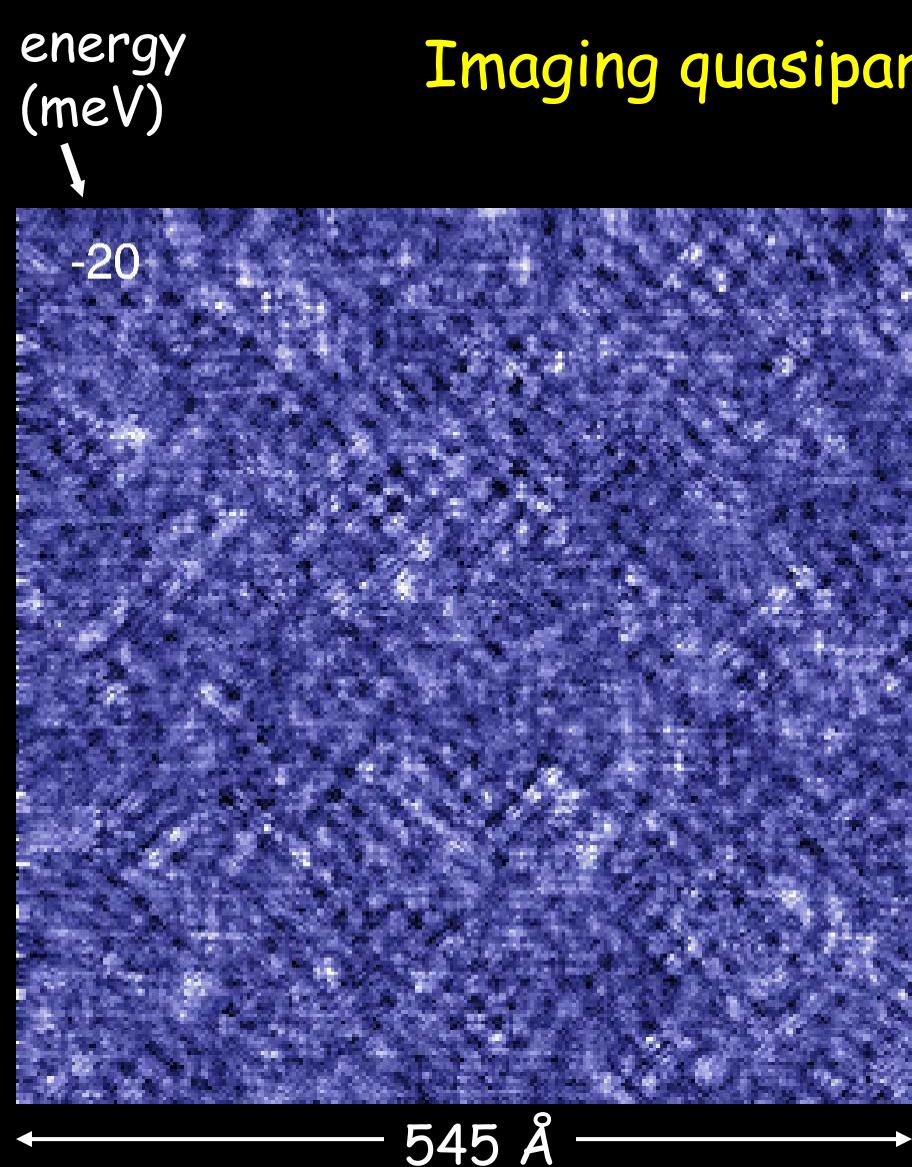


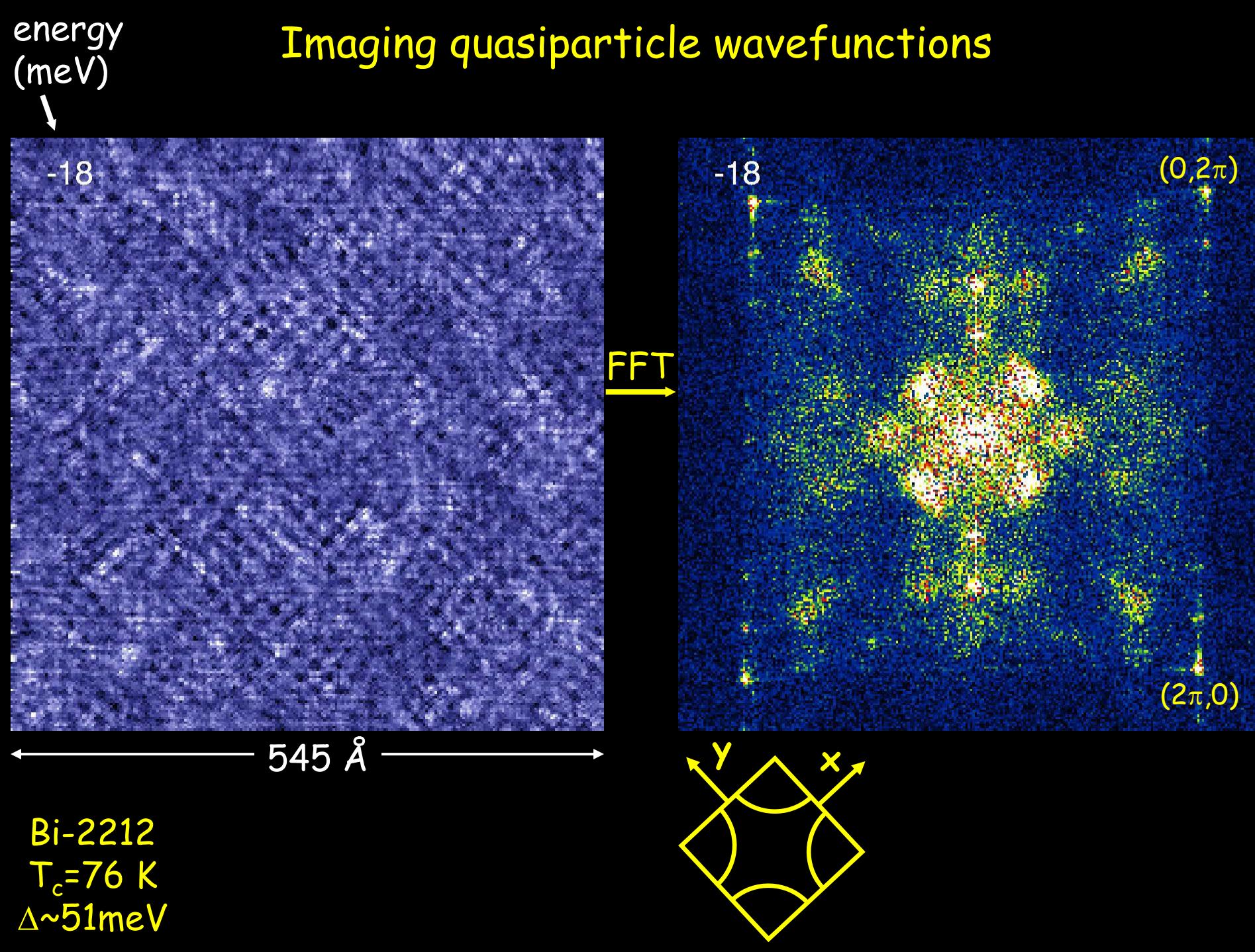
Bi-2212  
 $T_c=76$  K  
 $\Delta \sim 51$  meV



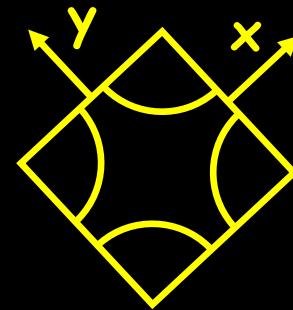
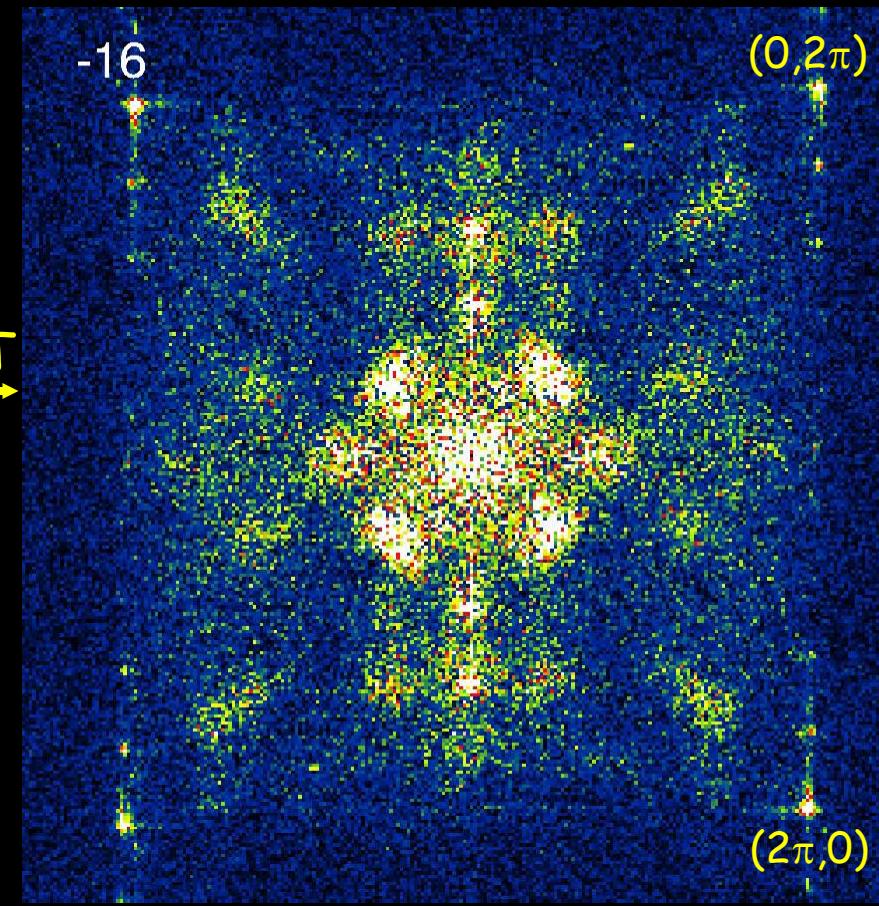
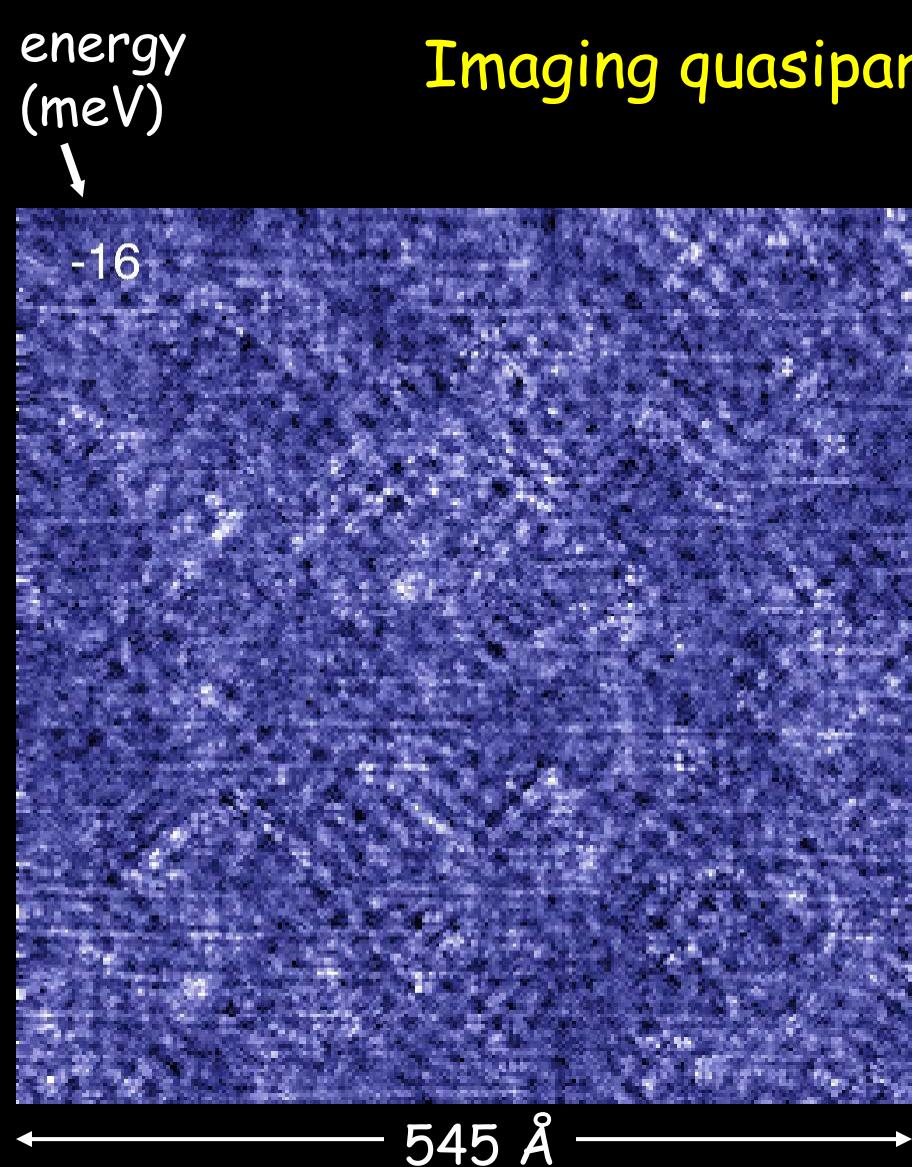


# Imaging quasiparticle wavefunctions

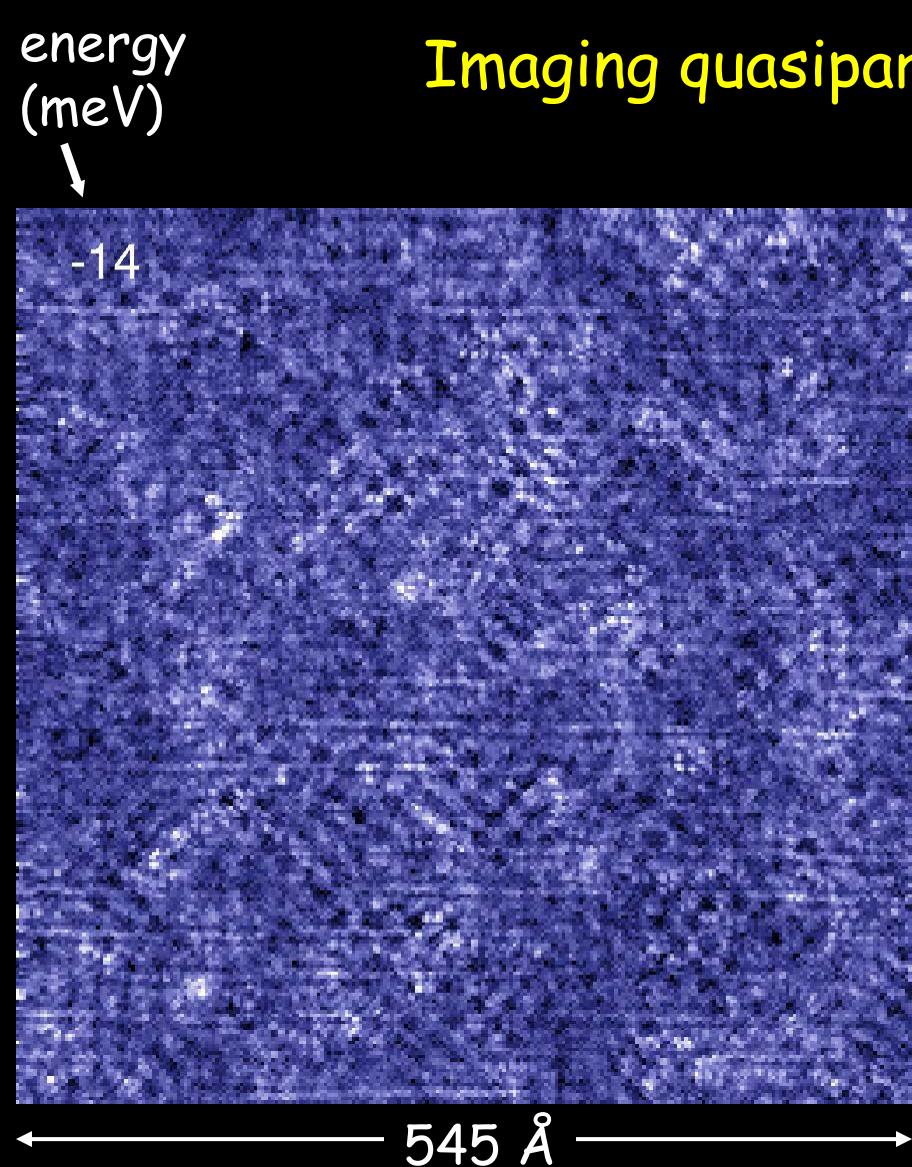




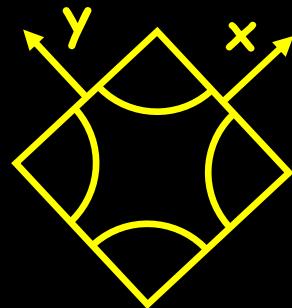
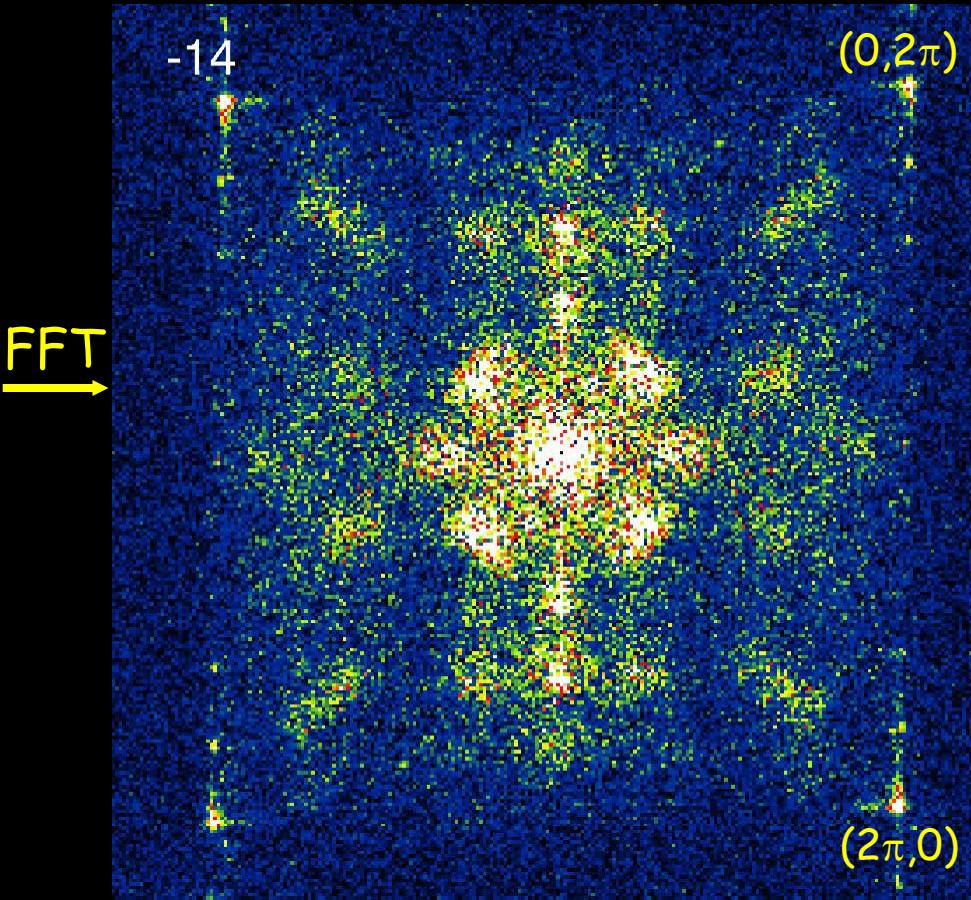
# Imaging quasiparticle wavefunctions



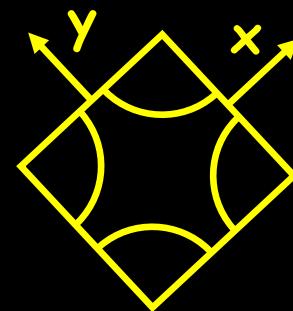
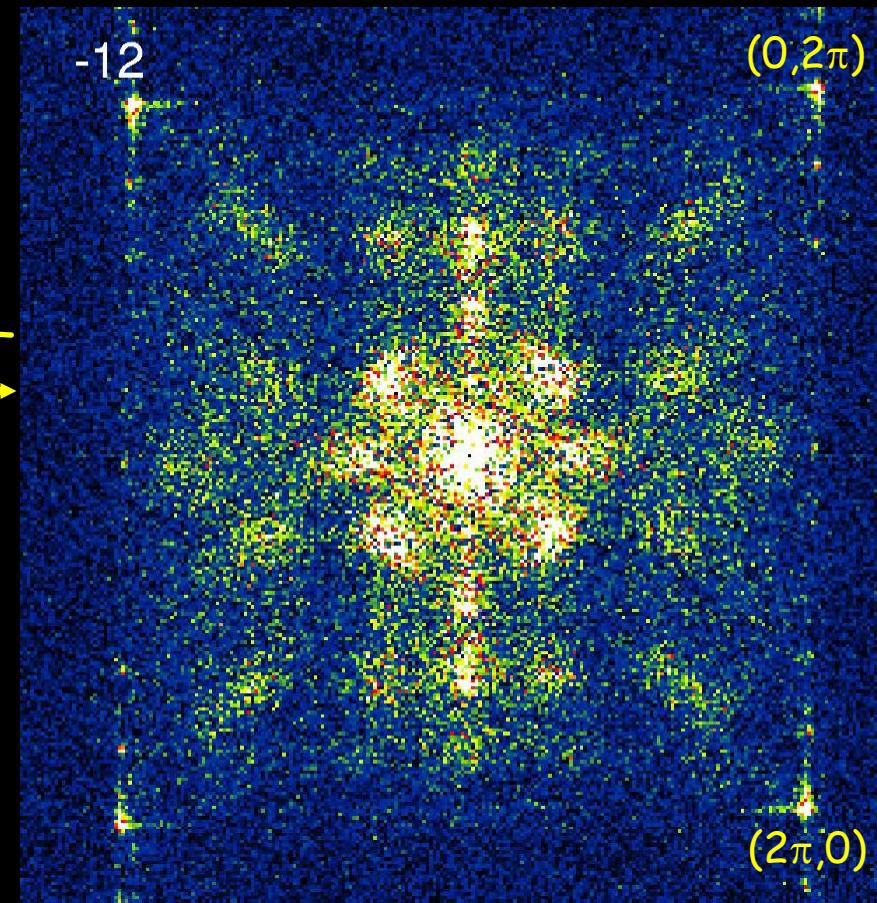
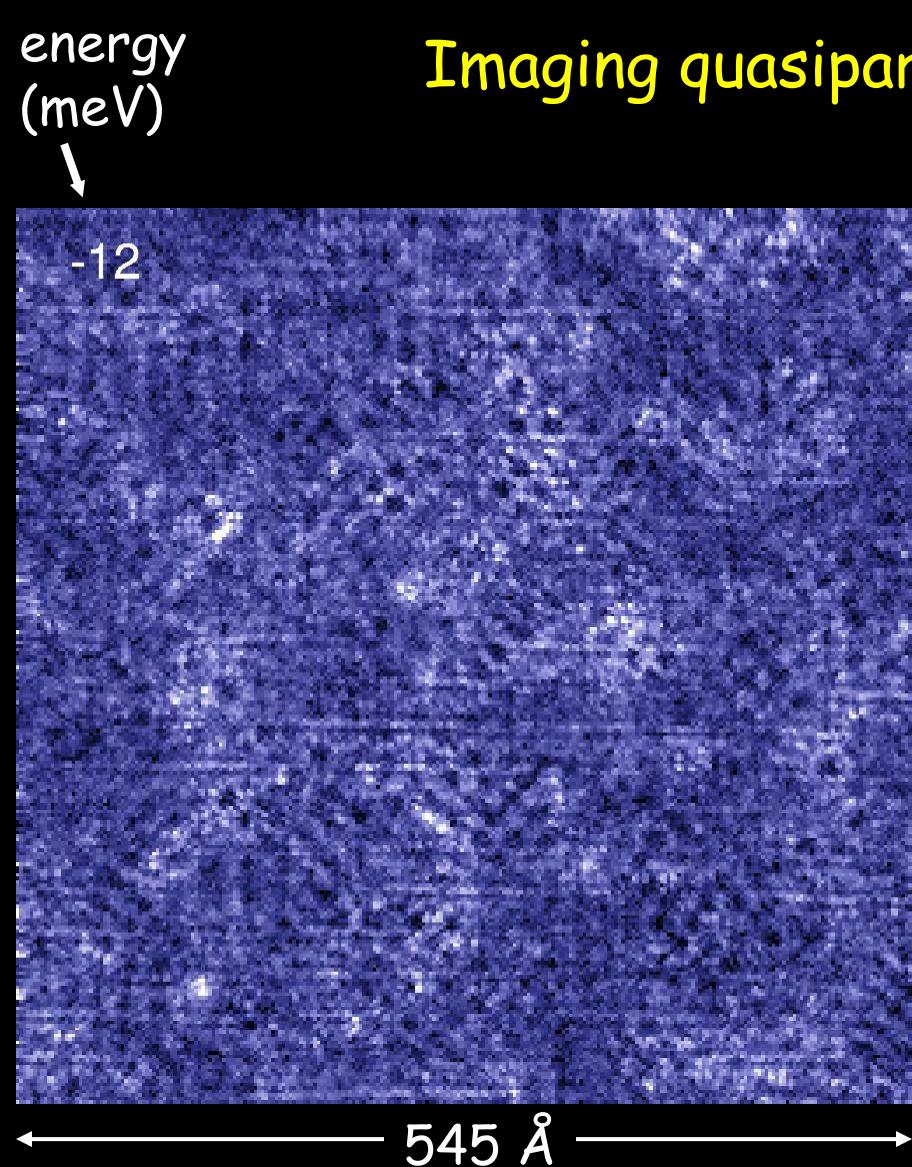
# Imaging quasiparticle wavefunctions



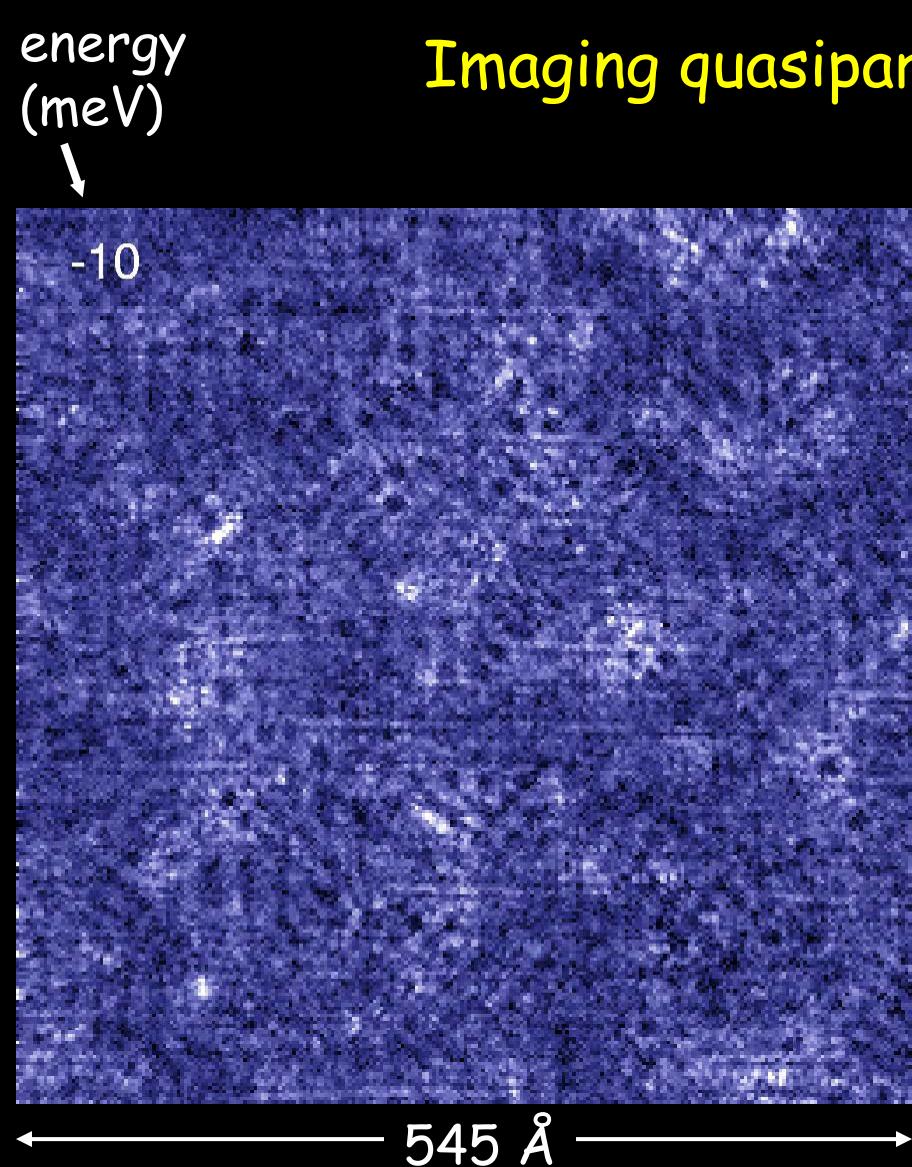
Bi-2212  
 $T_c=76$  K  
 $\Delta \sim 51$  meV



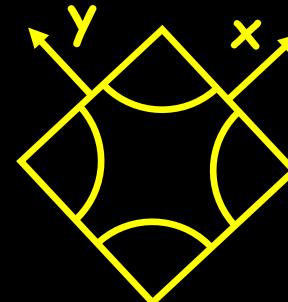
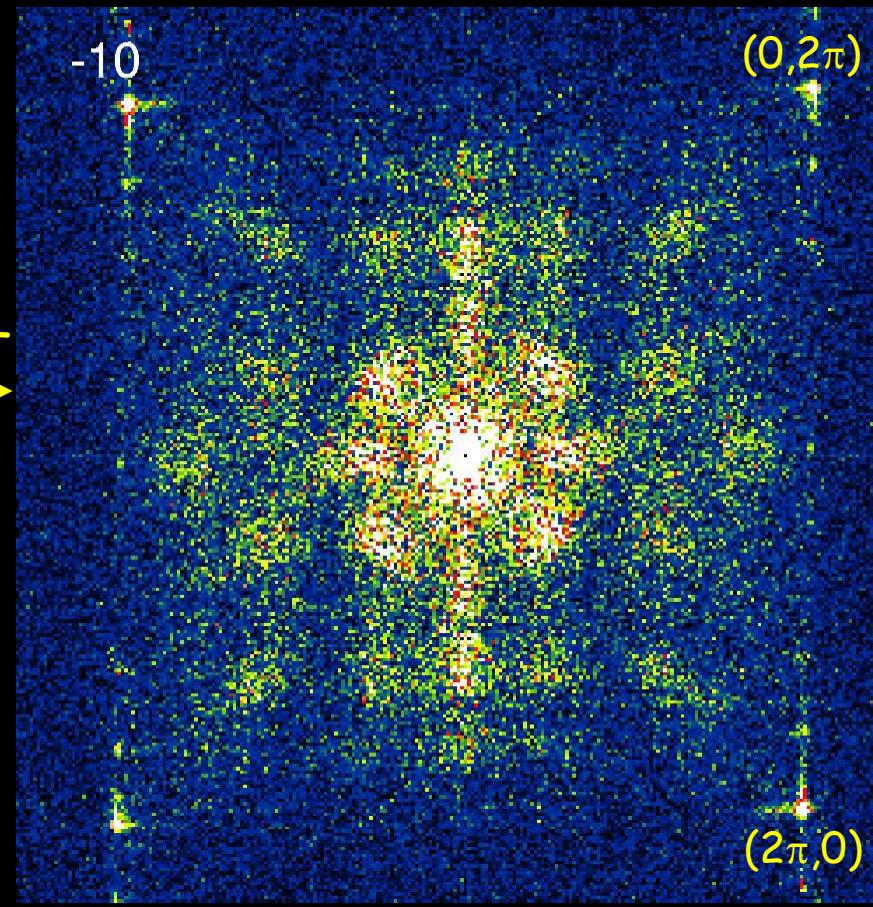
# Imaging quasiparticle wavefunctions



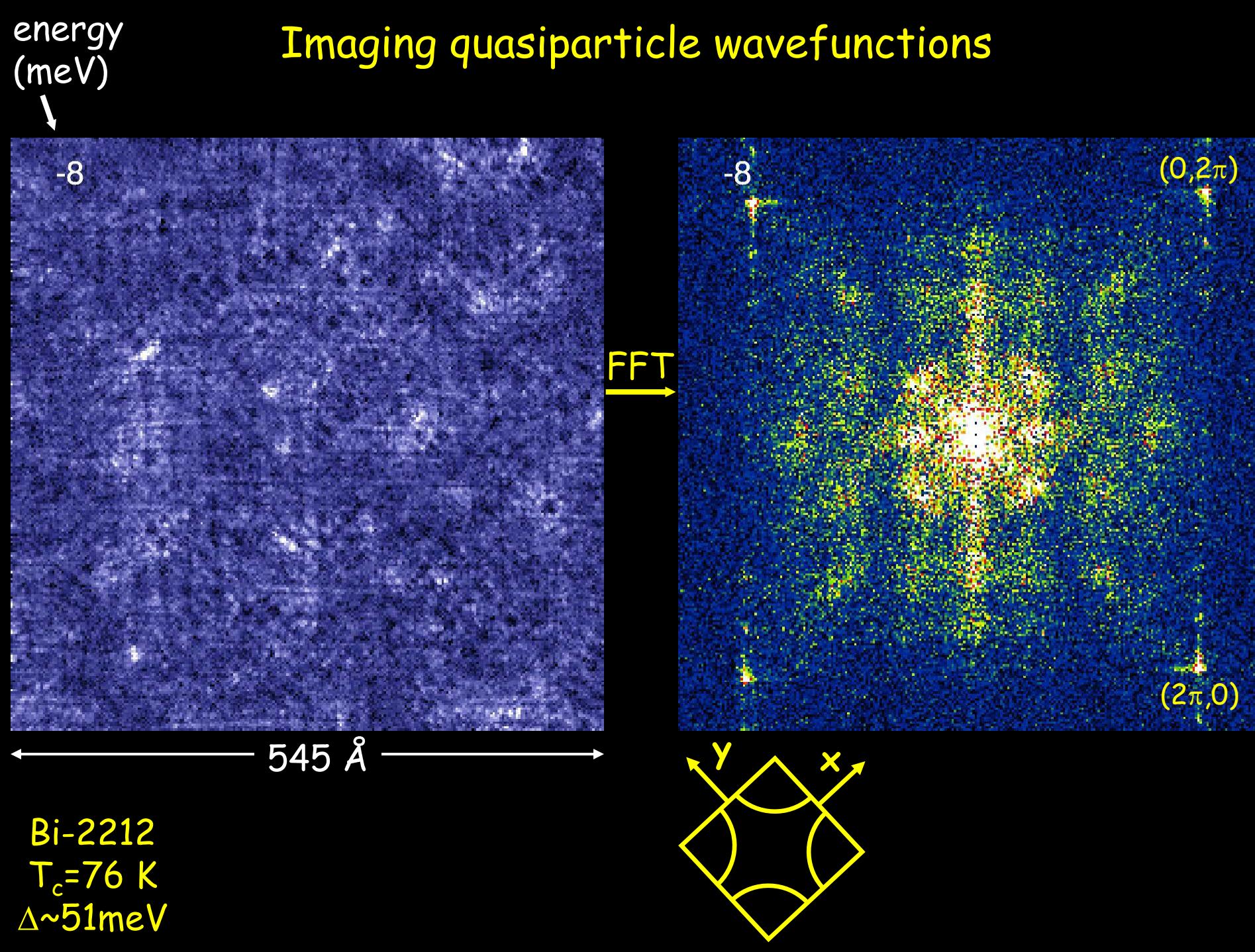
# Imaging quasiparticle wavefunctions



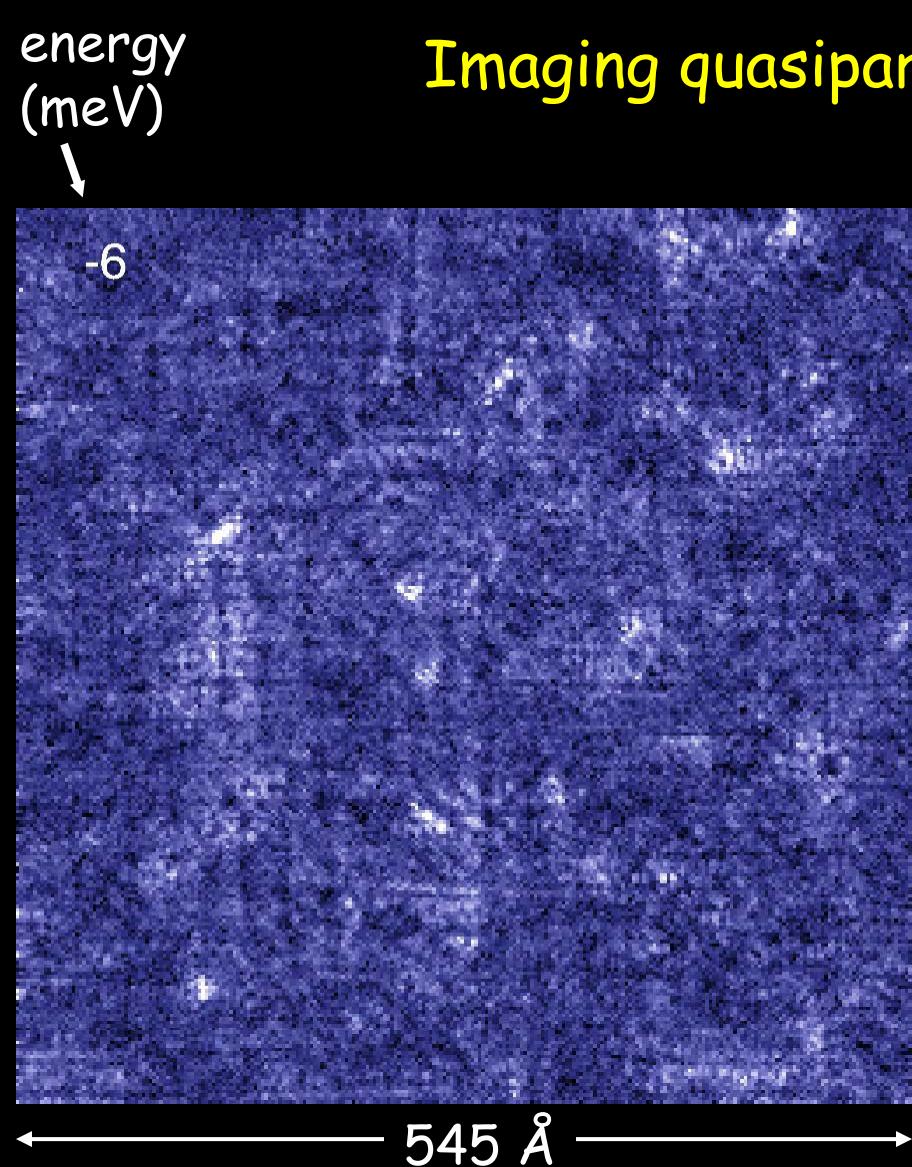
FFT



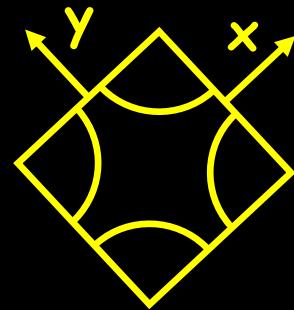
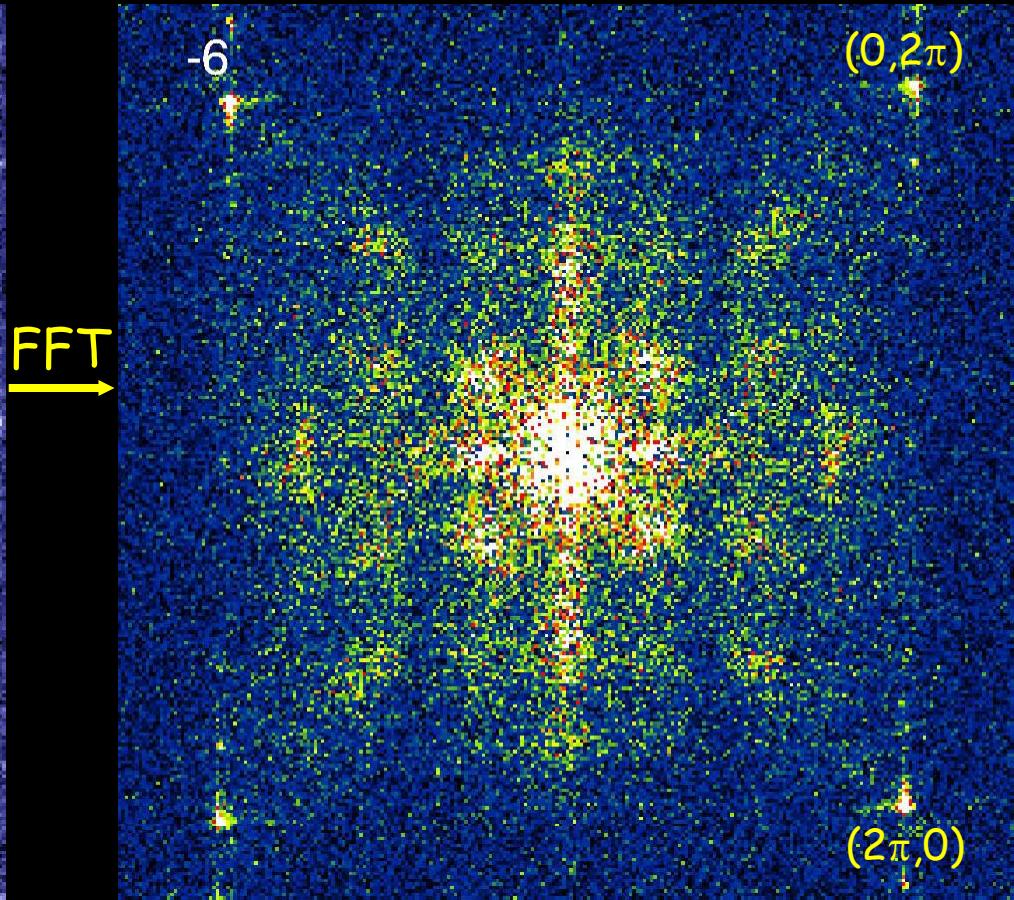
Bi-2212  
 $T_c=76$  K  
 $\Delta \sim 51$  meV



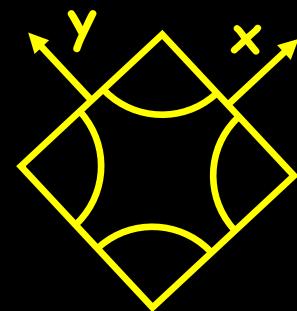
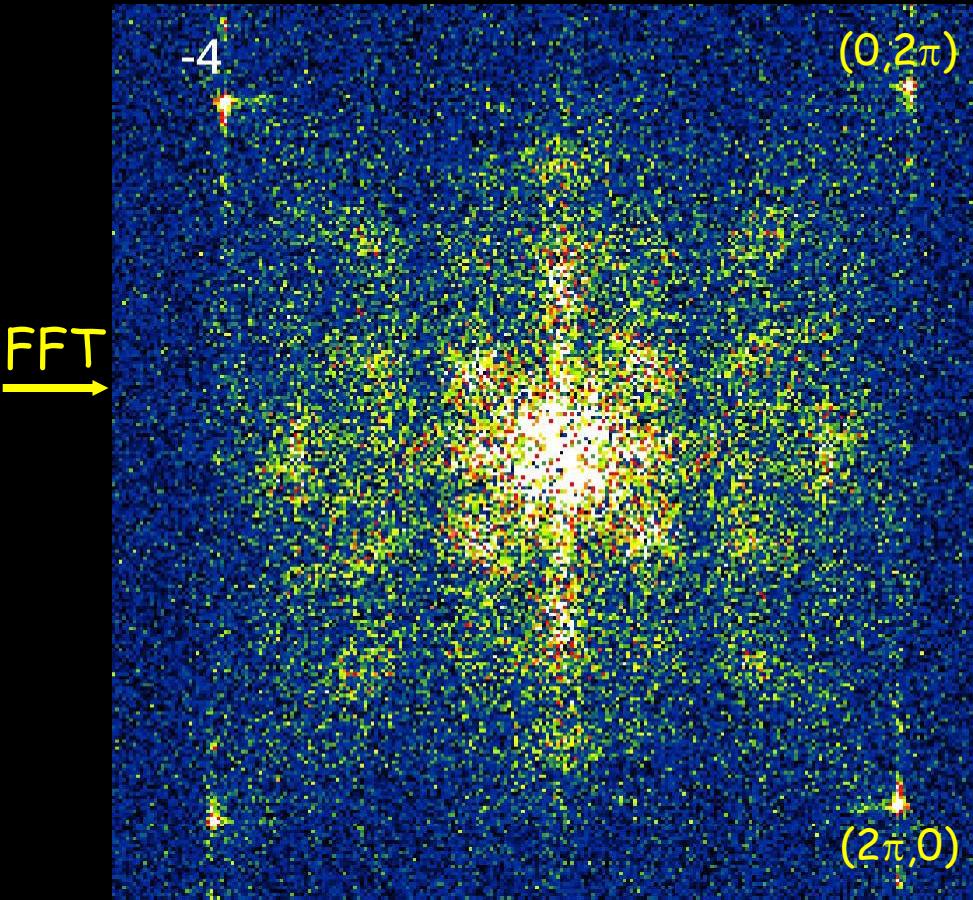
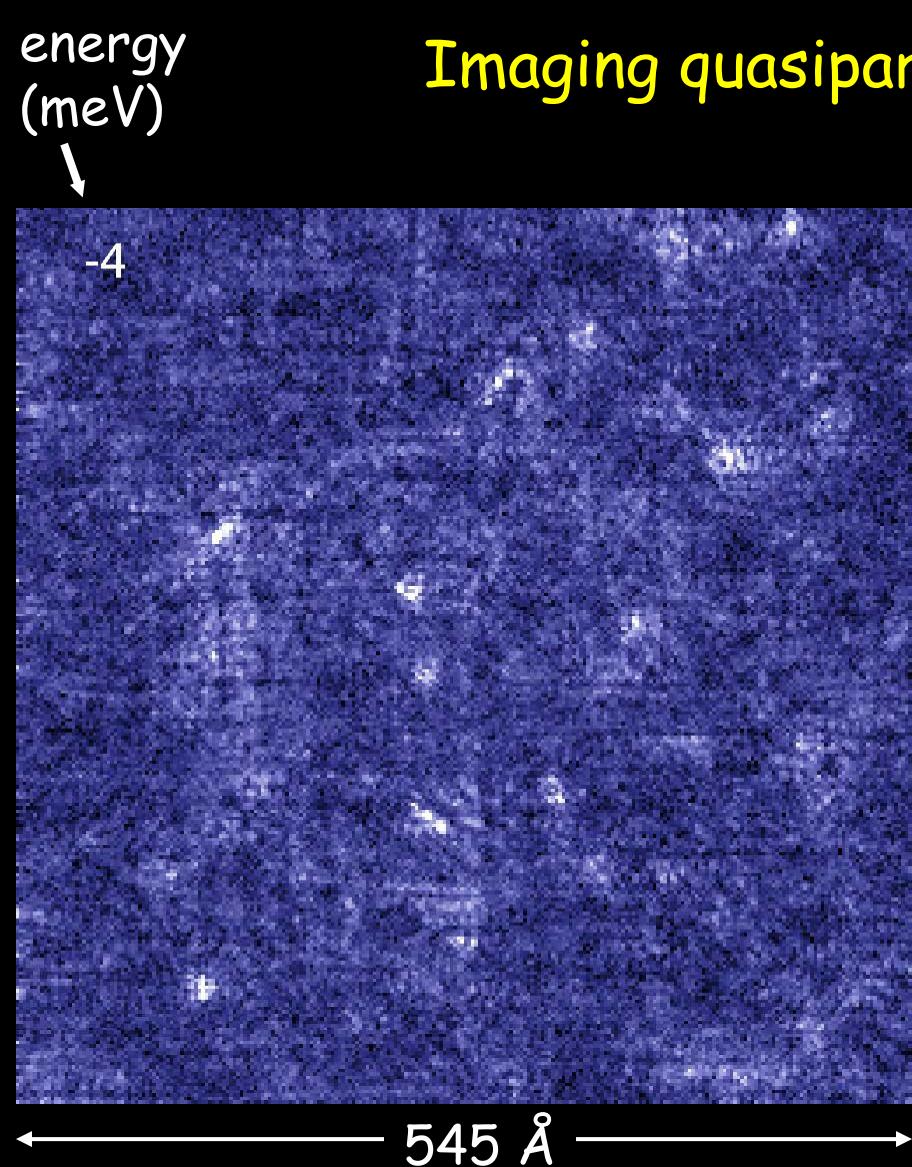
# Imaging quasiparticle wavefunctions



Bi-2212  
 $T_c=76$  K  
 $\Delta \sim 51$  meV



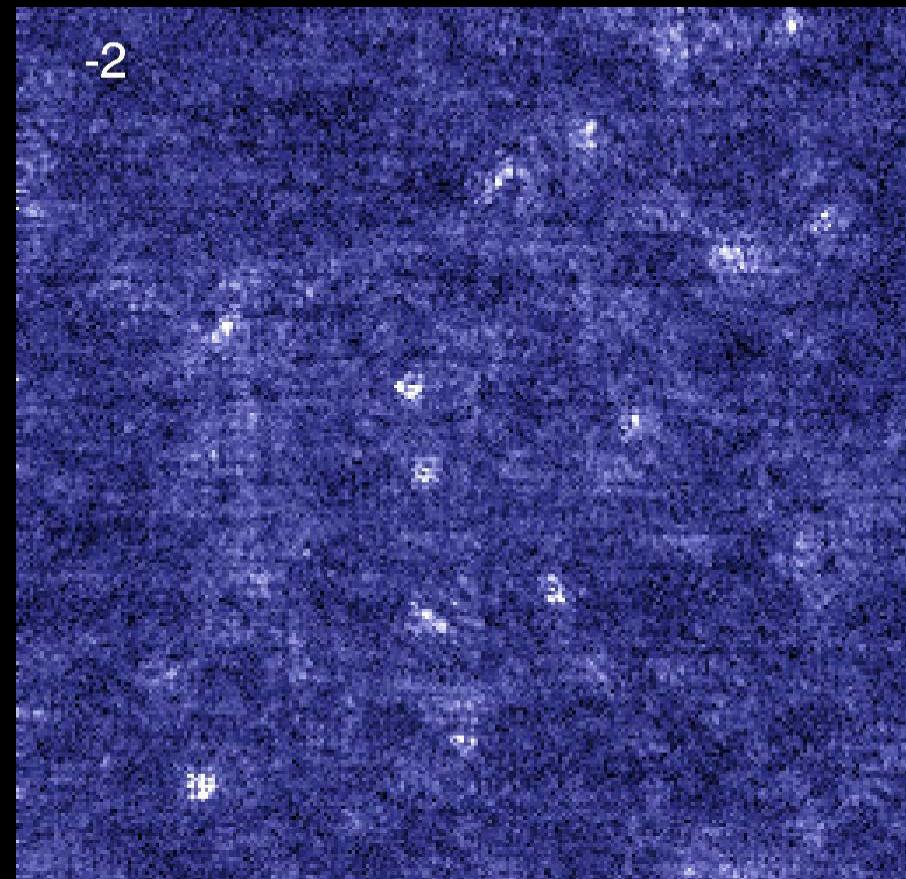
# Imaging quasiparticle wavefunctions



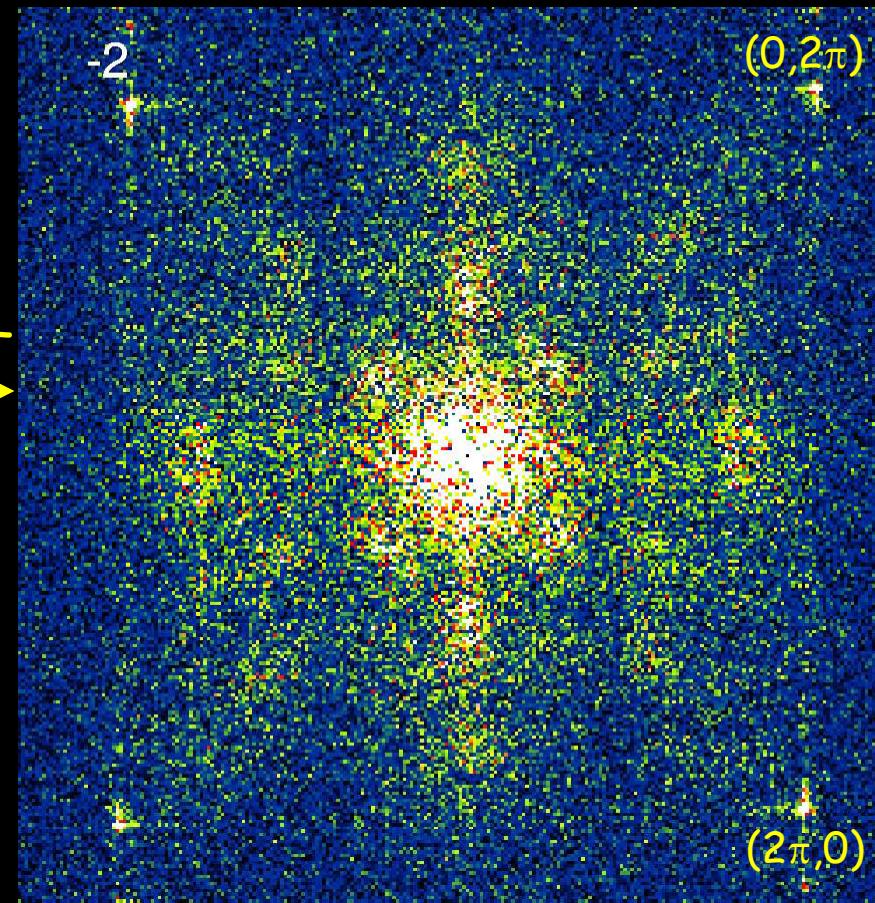
Bi-2212  
 $T_c=76$  K  
 $\Delta \sim 51$  meV

energy  
(meV)  
↓

# Imaging quasiparticle wavefunctions

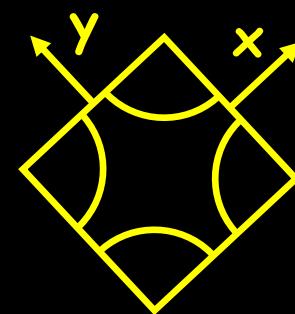


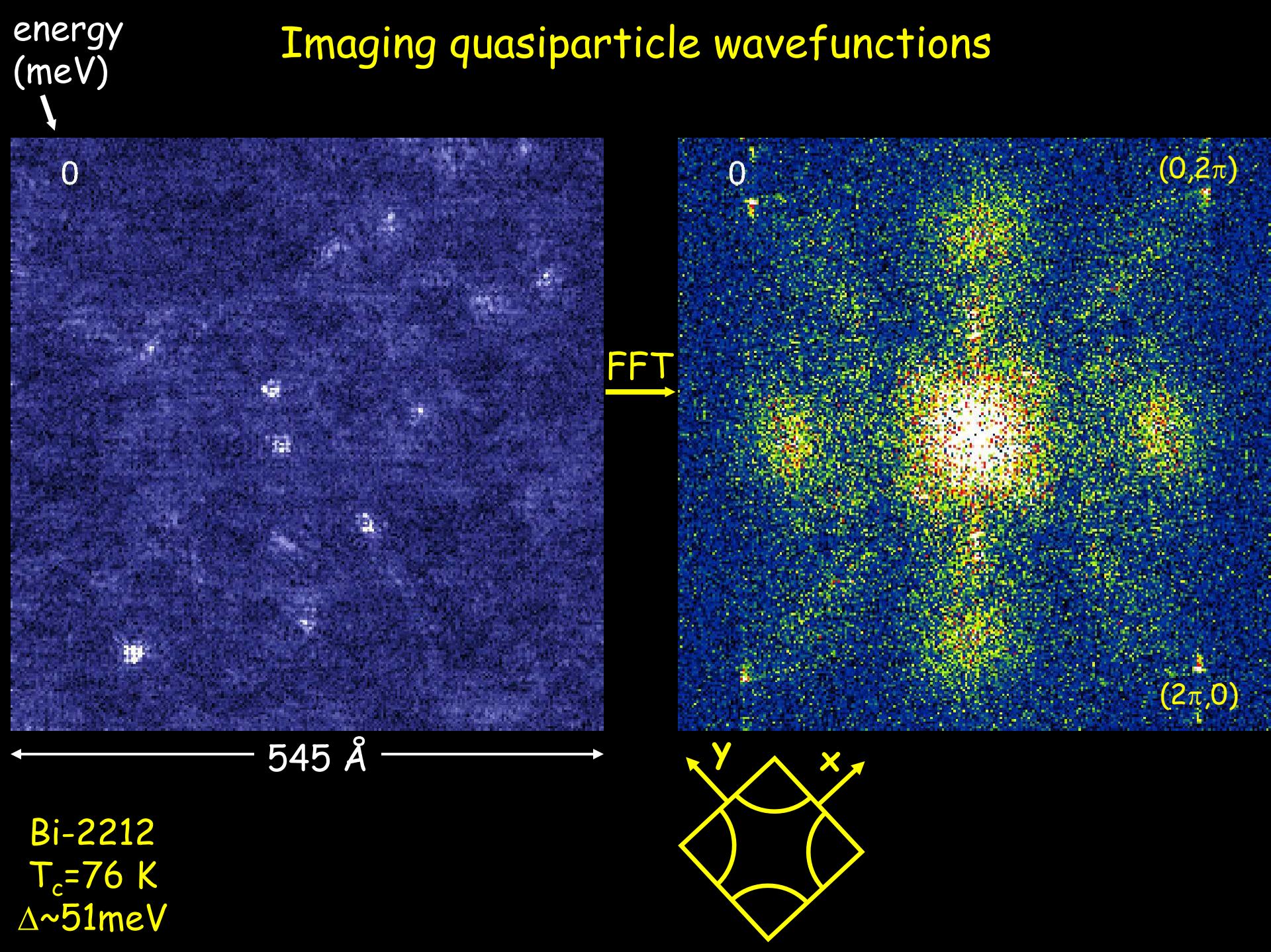
FFT



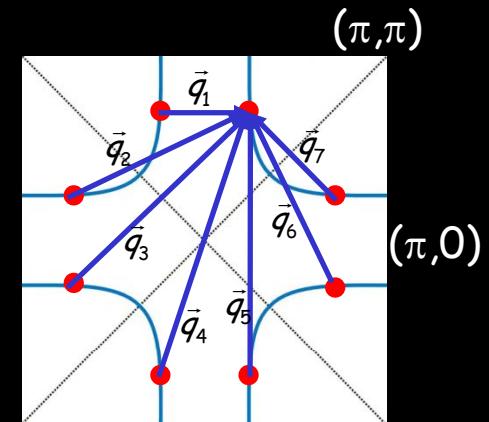
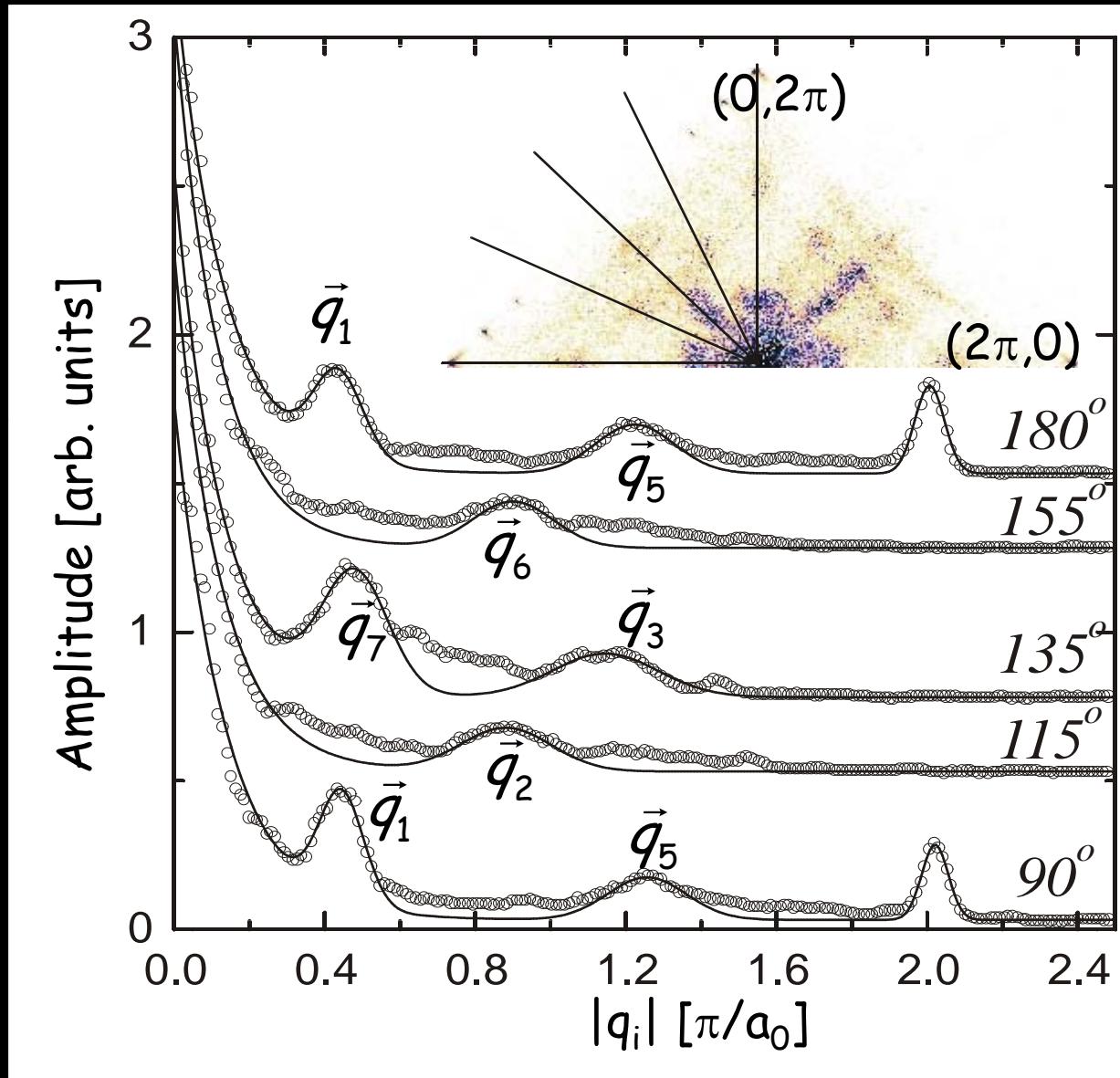
← 545 Å →

Bi-2212  
 $T_c = 76$  K  
 $\Delta \sim 51$  meV

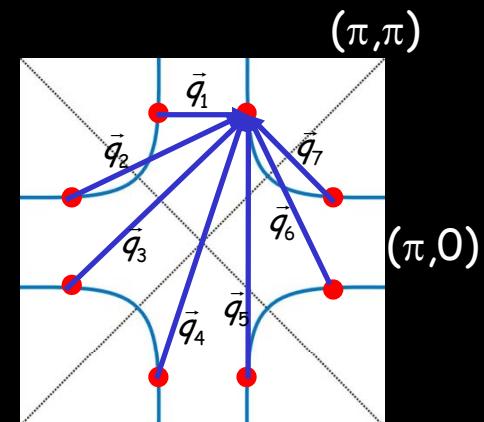
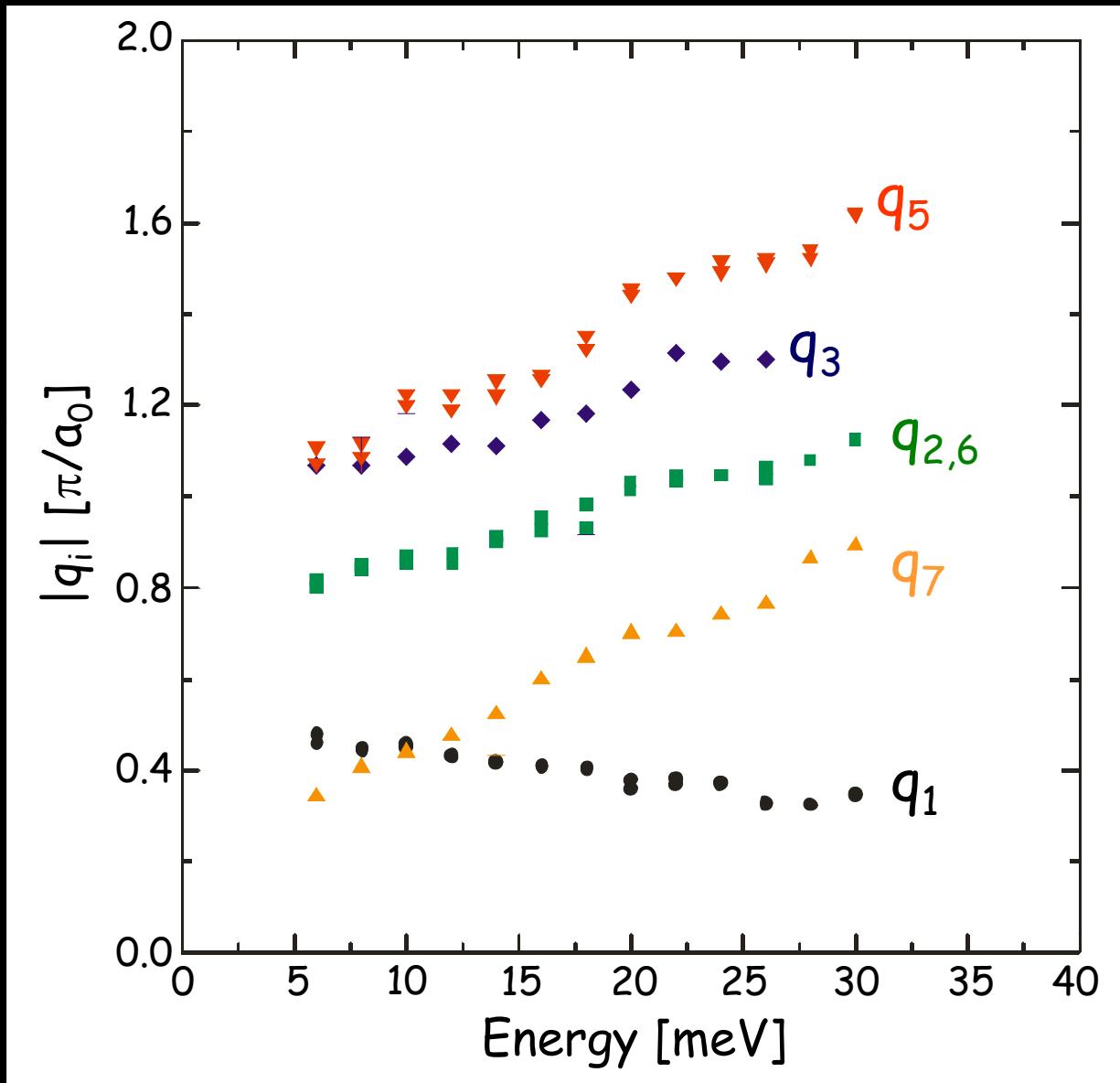




# Measuring the dispersion of the $q_i(E)$

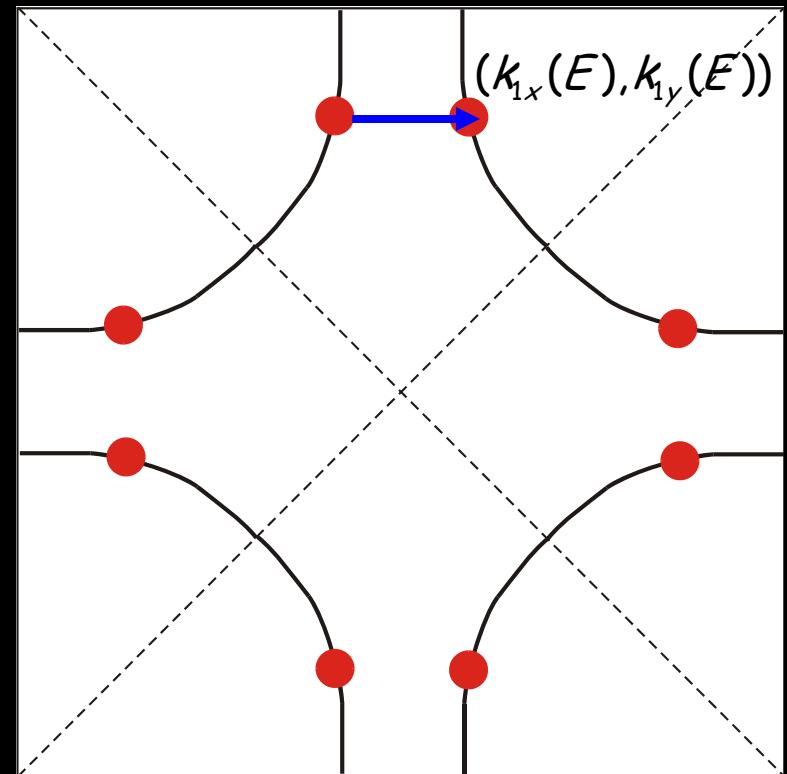


# Measuring the dispersion of the $q_i(E)$



# Measuring the Fermi Surface:

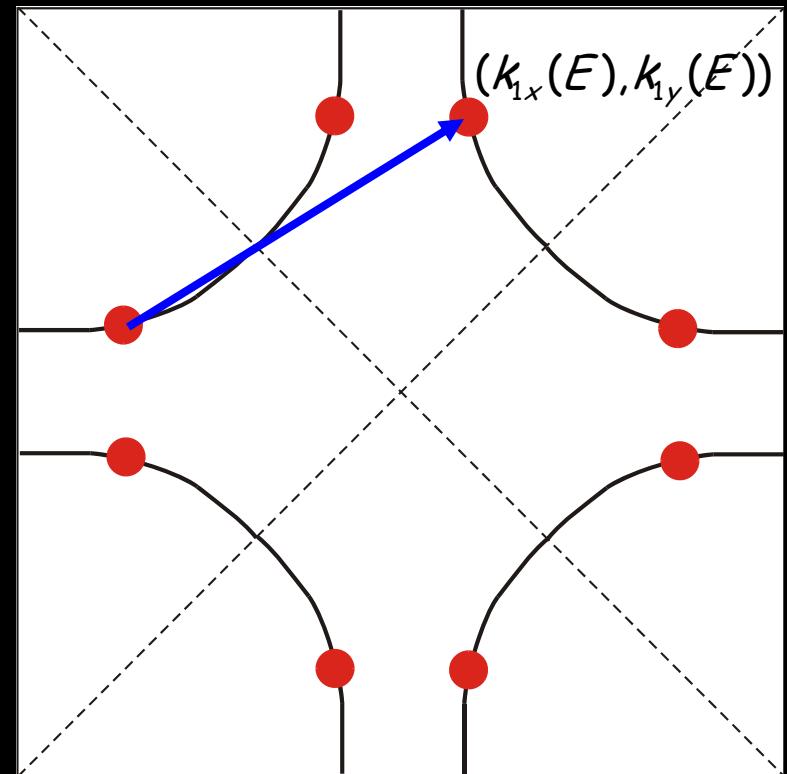
$$\vec{q}_1(E) = (2k_{1x}(E), 0)$$



## Measuring the Fermi Surface:

$$\vec{q}_1(E) = (2k_{1x}(E), 0)$$

$$\vec{q}_2(E) = (k_{1x}(E) + k_{1y}(E), k_{1y}(E) - k_{1x}(E))$$

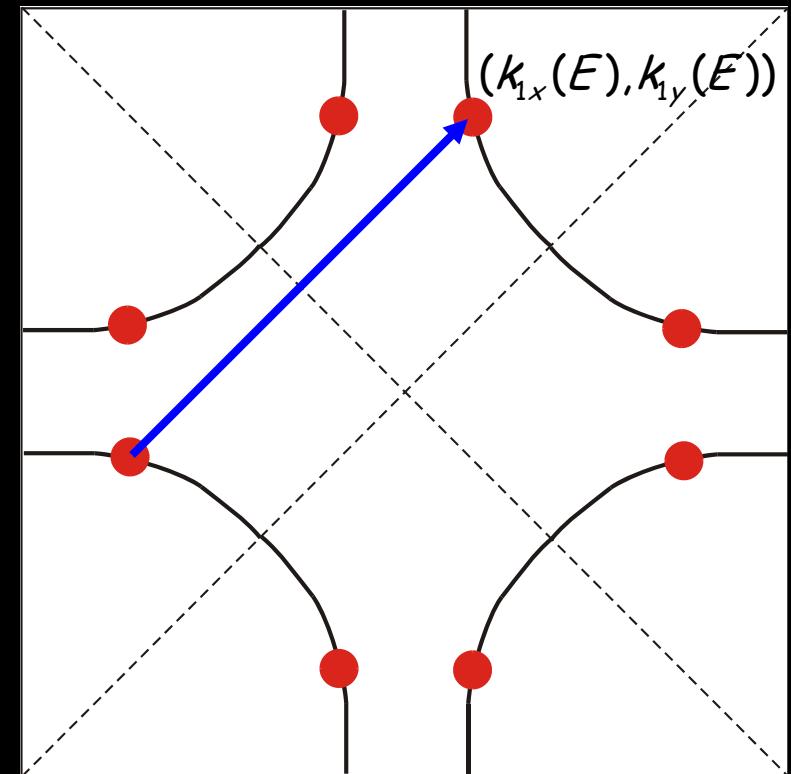


## Measuring the Fermi Surface:

$$\vec{q}_1(E) = (2k_{1x}(E), 0)$$

$$\vec{q}_2(E) = (k_{1x}(E) + k_{1y}(E), k_{1y}(E) - k_{1x}(E))$$

$$\vec{q}_3(E) = (k_{1x}(E) + k_{1y}(E), k_{1x}(E) + k_{1y}(E))$$



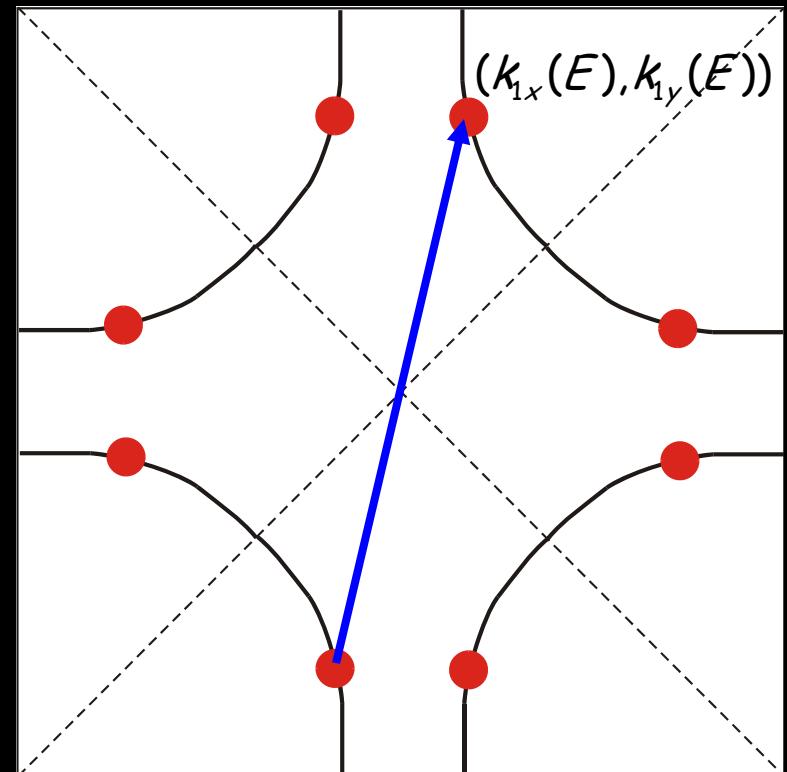
## Measuring the Fermi Surface:

$$\vec{q}_1(E) = (2k_{1x}(E), 0)$$

$$\vec{q}_2(E) = (k_{1x}(E) + k_{1y}(E), k_{1y}(E) - k_{1x}(E))$$

$$\vec{q}_3(E) = (k_{1x}(E) + k_{1y}(E), k_{1x}(E) + k_{1y}(E))$$

$$\vec{q}_4(E) = (2k_{1x}(E), 2k_{1y}(E))$$



## Measuring the Fermi Surface:

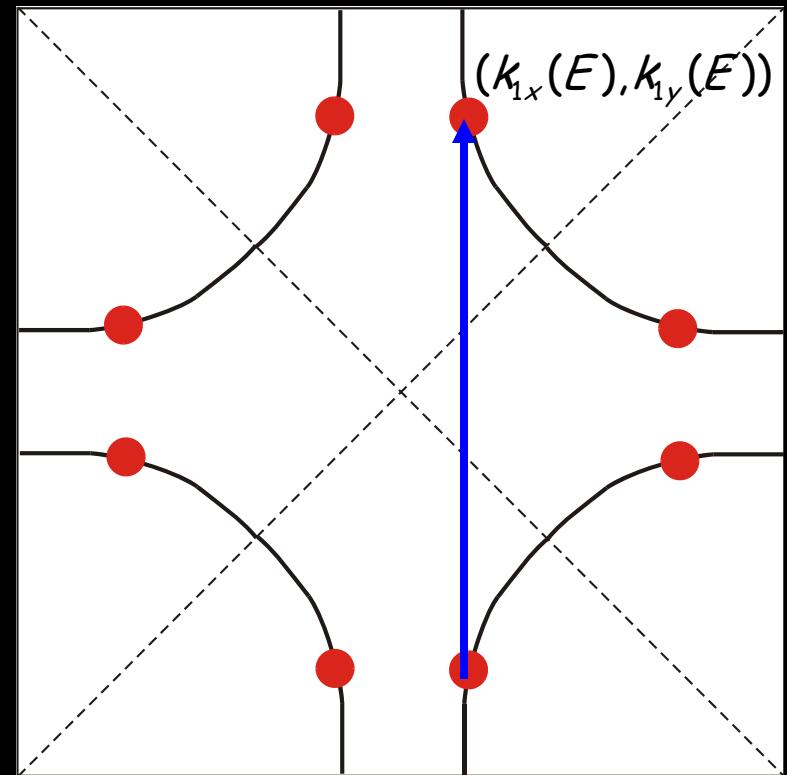
$$\vec{q}_1(E) = (2k_{1x}(E), 0)$$

$$\vec{q}_2(E) = (k_{1x}(E) + k_{1y}(E), k_{1y}(E) - k_{1x}(E))$$

$$\vec{q}_3(E) = (k_{1x}(E) + k_{1y}(E), k_{1x}(E) + k_{1y}(E))$$

$$\vec{q}_4(E) = (2k_{1x}(E), 2k_{1y}(E))$$

$$\vec{q}_5(E) = (0, 2k_{1y}(E))$$



## Measuring the Fermi Surface:

$$\vec{q}_1(E) = (2k_{1x}(E), 0)$$

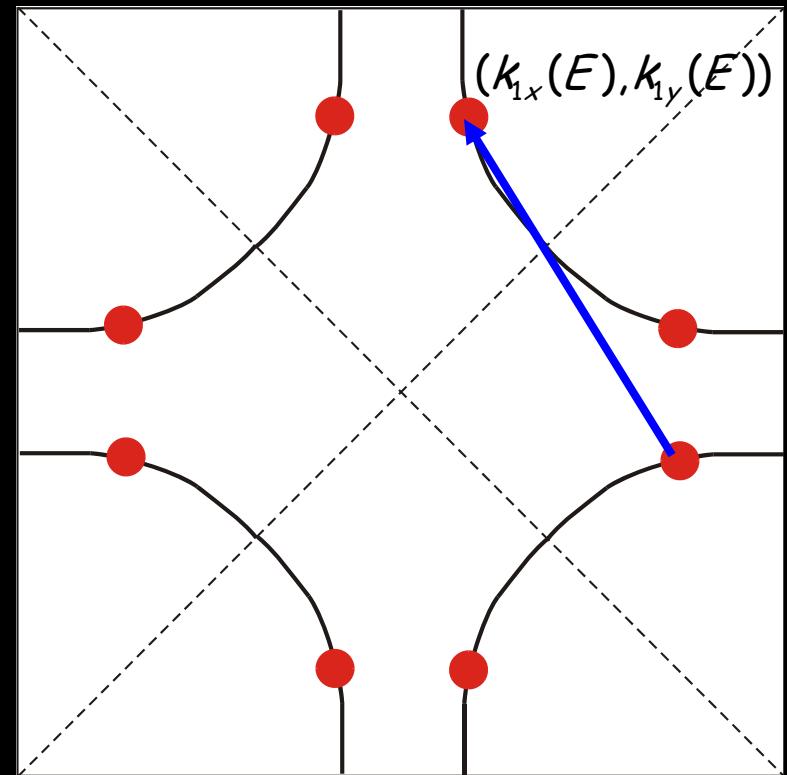
$$\vec{q}_2(E) = (k_{1x}(E) + k_{1y}(E), k_{1y}(E) - k_{1x}(E))$$

$$\vec{q}_3(E) = (k_{1x}(E) + k_{1y}(E), k_{1x}(E) + k_{1y}(E))$$

$$\vec{q}_4(E) = (2k_{1x}(E), 2k_{1y}(E))$$

$$\vec{q}_5(E) = (0, 2k_{1y}(E))$$

$$\vec{q}_6(E) = (k_{1x}(E) - k_{1y}(E), k_{1y}(E) + k_{1x}(E))$$



# Measuring the Fermi Surface:

$$\vec{q}_1(E) = (2k_{1x}(E), 0)$$

$$\vec{q}_2(E) = (k_{1x}(E) + k_{1y}(E), k_{1y}(E) - k_{1x}(E))$$

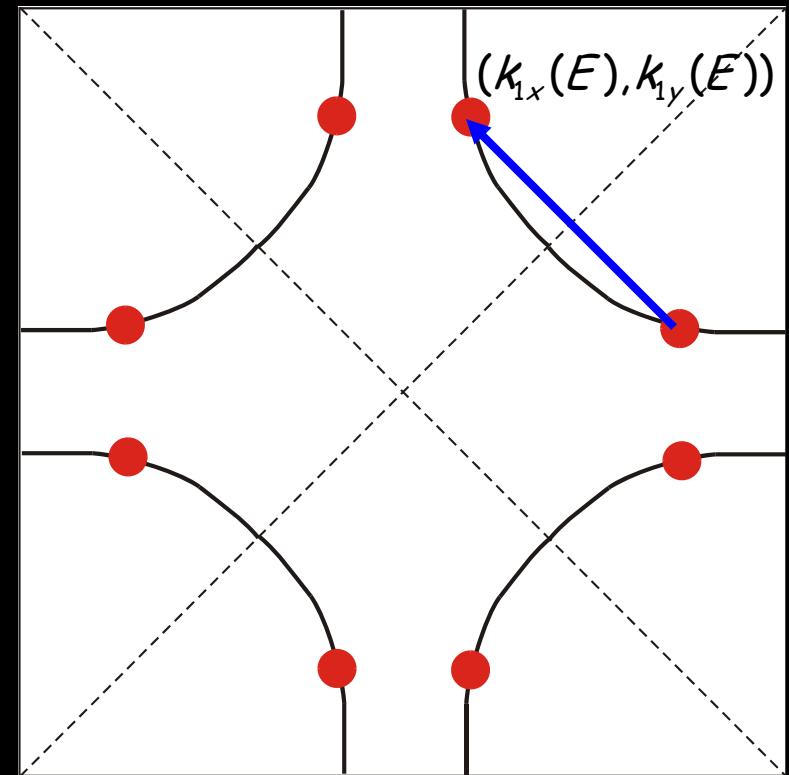
$$\vec{q}_3(E) = (k_{1x}(E) + k_{1y}(E), k_{1x}(E) + k_{1y}(E))$$

$$\vec{q}_4(E) = (2k_{1x}(E), 2k_{1y}(E))$$

$$\vec{q}_5(E) = (0, 2k_{1y}(E))$$

$$\vec{q}_6(E) = (k_{1x}(E) - k_{1y}(E), k_{1y}(E) + k_{1x}(E))$$

$$\vec{q}_7(E) = (k_{1x}(E) - k_{1y}(E), k_{1y}(E) - k_{1x}(E))$$

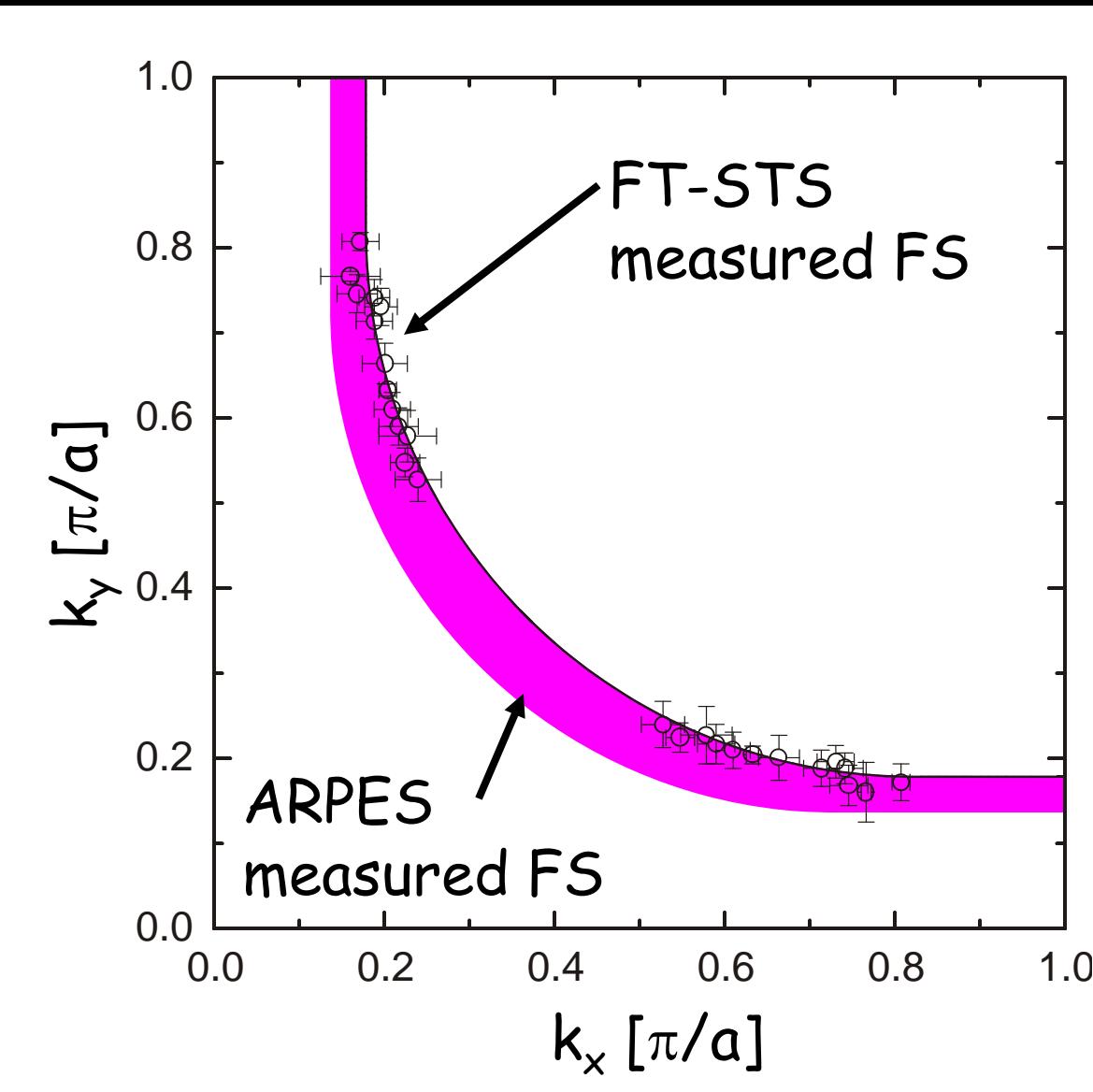


For each energy:

- 7 measured quantities
- 7 equations
- 2 unknowns ( $k_x, k_y$ )

System is vastly overdetermined!

# ARPES & STM: Fermi surface comparison

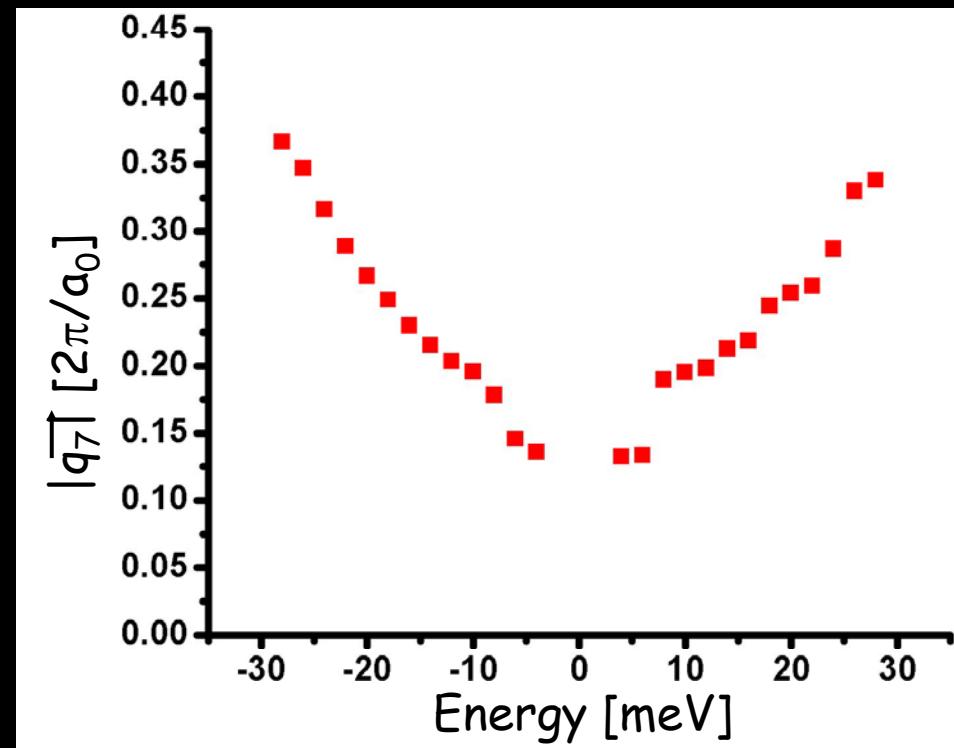
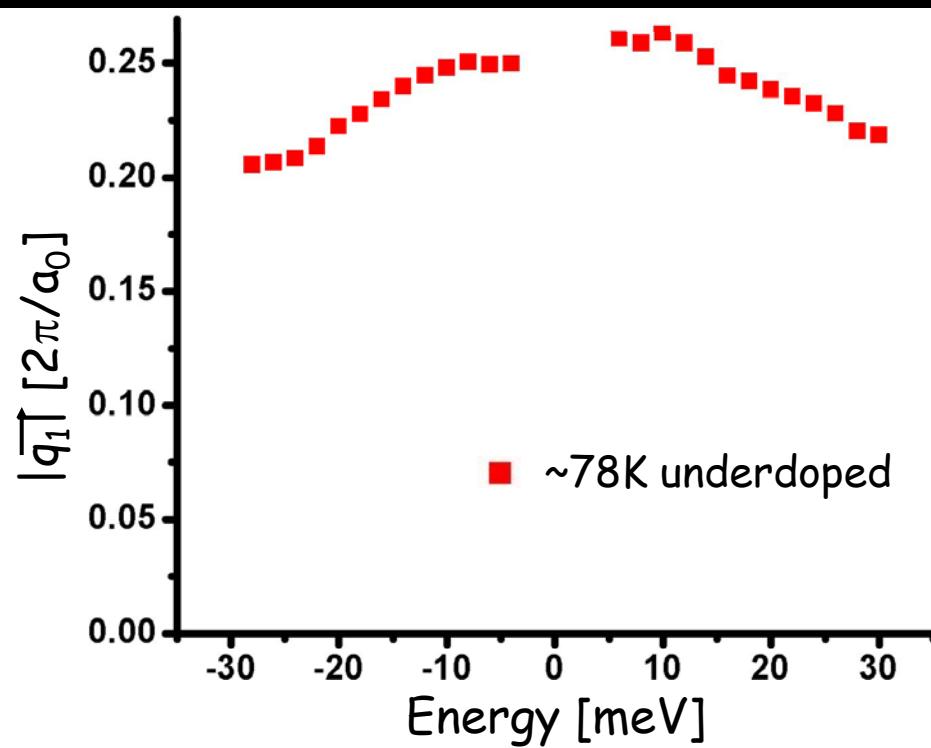
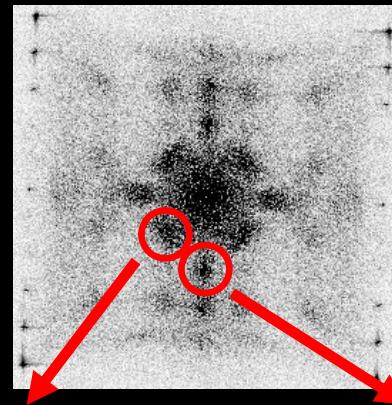


What about doping dependence?

# Dispersion of $q$ -space peaks

$(0,\pi)$  anti-nodal  
*Cu-O-Cu bond*

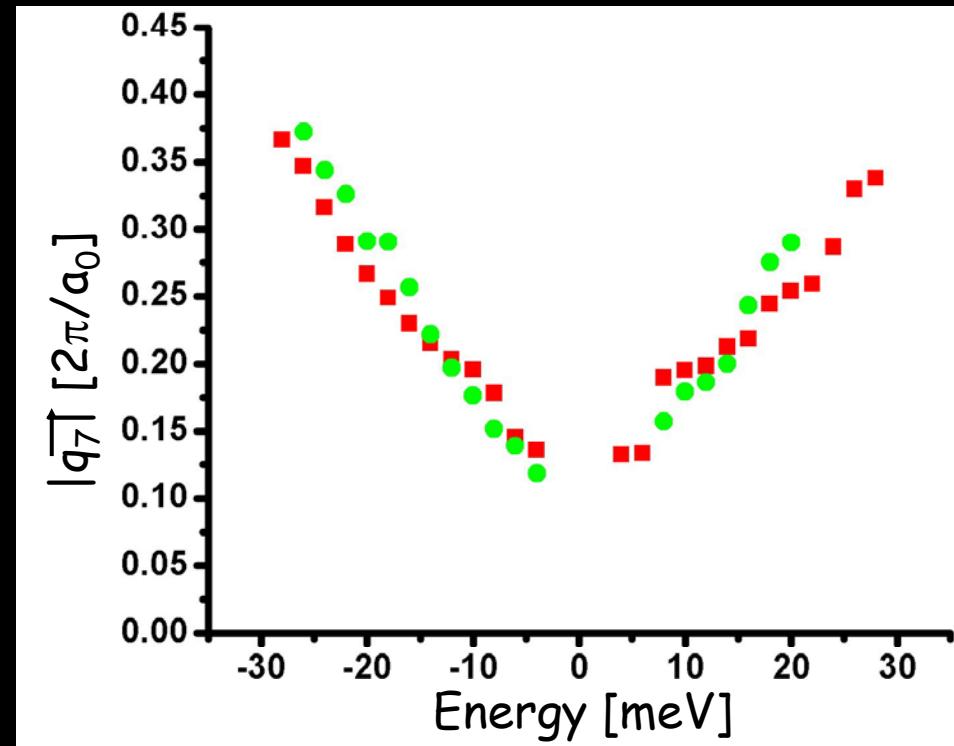
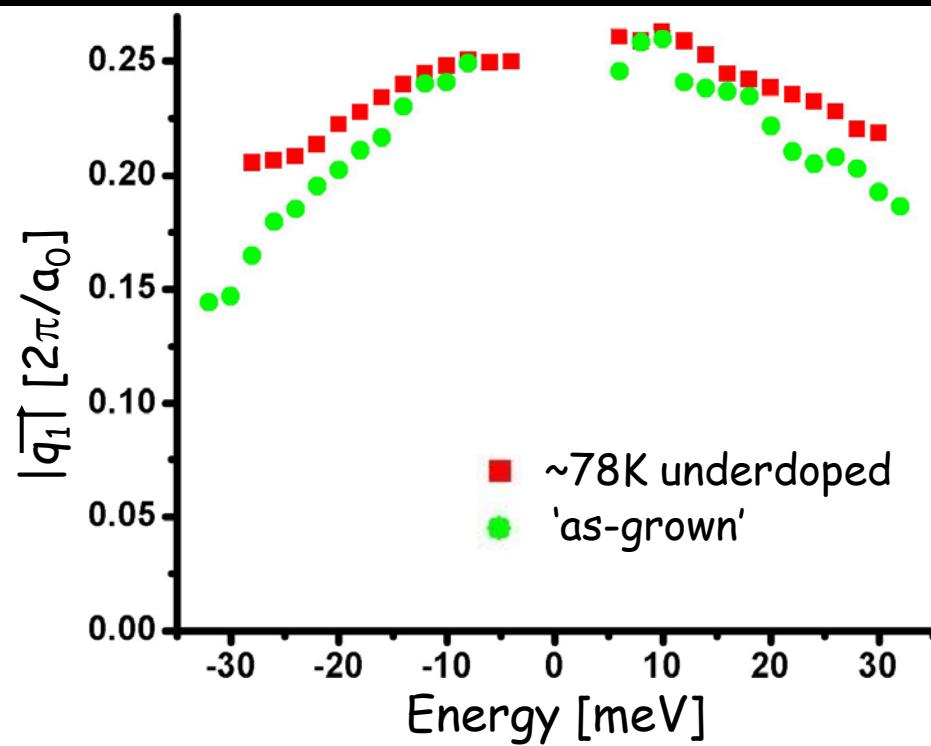
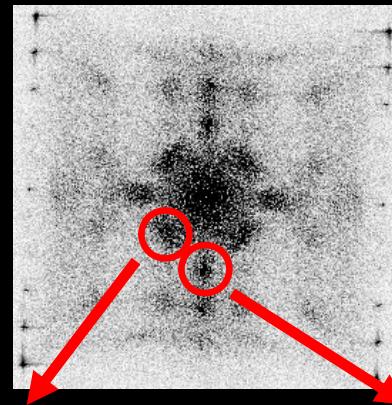
$(\pi,\pi)$  nodal  
45° from bond



# Dispersion of $q$ -space peaks

$(0,\pi)$  anti-nodal  
*Cu-O-Cu bond*

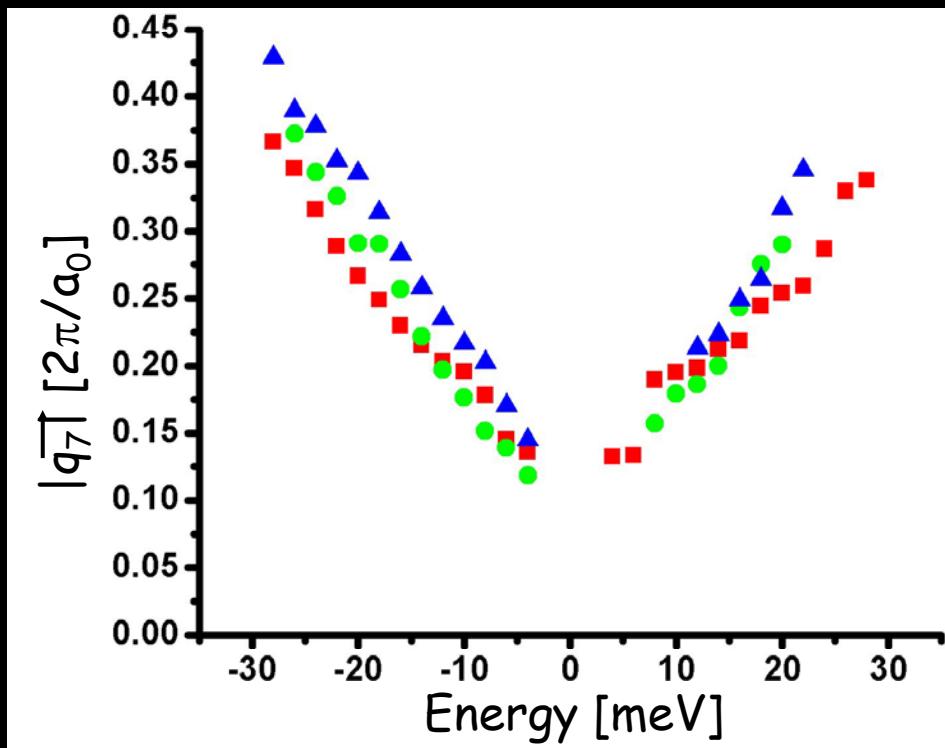
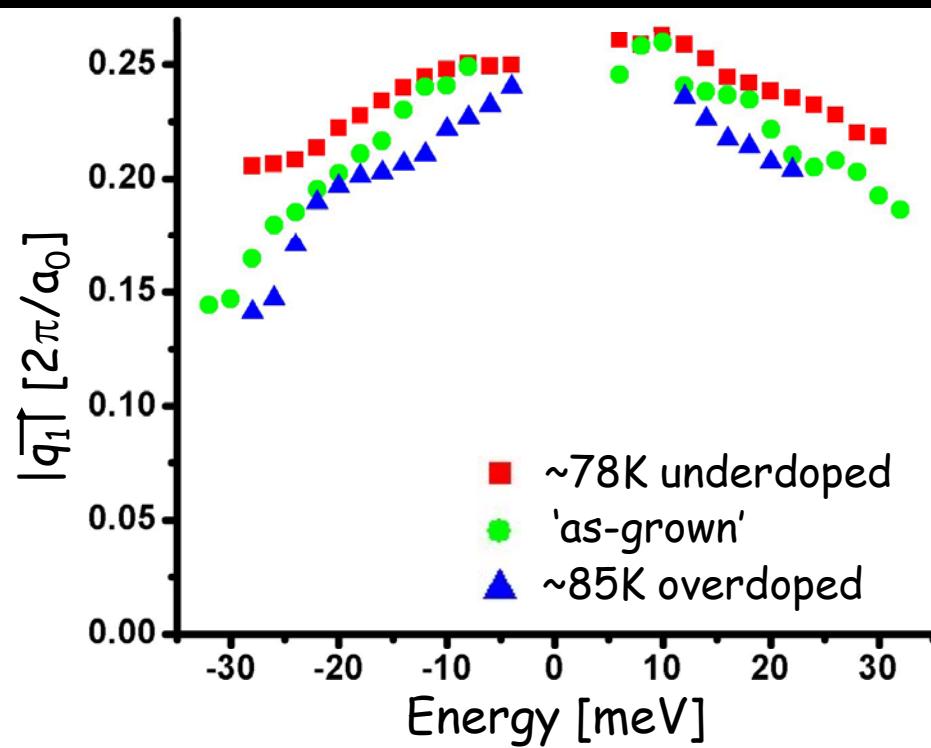
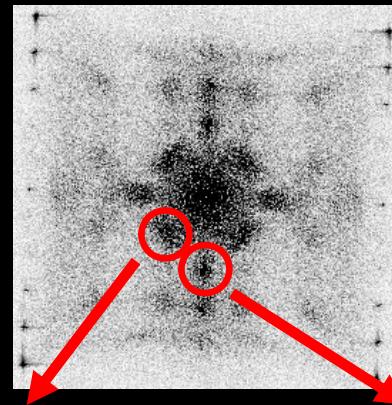
$(\pi,\pi)$  nodal  
45° from bond



# Dispersion of $q$ -space peaks

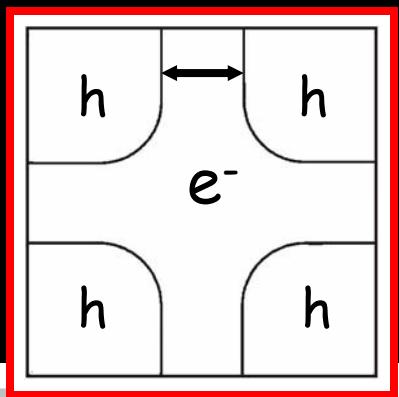
$(0,\pi)$  anti-nodal  
*Cu-O-Cu bond*

$(\pi,\pi)$  nodal  
45° from bond



# Doping trend fits expectation, agrees with ARPES

$(\pi, 0)$   
anti-nodal



$(\pi, \pi)$   
nodal

