Underdamped

$$q(t) = \mathrm{Re}\left[\frac{-iV_0/L}{\omega_0^2-\omega_d^2+i\Gamma\omega_d}\, e^{i\omega_d t}\right] + e^{-\Gamma t/2}\left[A\cos(\omega t)+B\sin(\omega t)\right]$$

$$= \frac{-\Gamma\omega_d V_0/L}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2}\cos(\omega_d t) + \frac{V_0/L\,(\omega_0^2-\omega_d^2)}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2}\sin(\omega_d t)$$

$$+ e^{-\Gamma t/2}\left[A\cos(\omega t)+B\sin(\omega t)\right]$$

$$q(t=0) = \frac{-\Gamma\omega_d V_0/L}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2} + A = q_0$$

$$I(t) = \frac{\Gamma\omega\omega_d V_0/L}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2}\sin(\omega_d t) + \frac{\omega_d V_0/L\,(\omega_0^2-\omega_d^2)}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2}\cos(\omega_d t)$$

$$-\frac{\Gamma}{2}e^{-\Gamma t/2}\left[A\cos(\omega t)+B\sin(\omega t)\right] + e^{-\Gamma t/2}\left[-\omega A\sin(\omega t)+\omega B\cos(\omega t)\right]$$

$$I(t=0) = \frac{\omega_d V_0/L\,(\omega_0^2-\omega_d^2)}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2} - \frac{\Gamma}{2}A + \omega B = I_0$$

$$\Rightarrow A = \frac{\Gamma\omega_d V_0/L}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2} + q_0$$

$$\Rightarrow B = \frac{1}{\omega}\left[I_0 + \frac{\Gamma}{2}A - \frac{\omega_d V_0/L\,(\omega_0^2-\omega_d^2)}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2}\right]$$

$$= \frac{I_0}{\omega} + \frac{\Gamma}{2\omega}q_0 + \frac{\omega_d V_0}{L}\,\frac{\Gamma^2/2-(\omega_0^2-\omega_d^2)}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2}$$

Overdamped

$$q(t) = Re\left[\frac{-iV_0/L}{\omega_0^2-\omega_d^2+i\Gamma\omega_d} e^{i\omega_d t}\right] + Ae^{-\left(\frac{\Gamma}{2}-\omega\right)t} + Be^{-\left(\frac{\Gamma}{2}+\omega\right)t}$$

$$= \frac{-\Gamma\omega_d V_0/L}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2}\cos(\omega_d t) + \frac{V_0/L\,(\omega_0^2-\omega_d^2)}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2}\sin(\omega_d t)$$

$$+ Ae^{-\left(\frac{\Gamma}{2}-\omega\right)t} + Be^{-\left(\frac{\Gamma}{2}+\omega\right)t}$$

$$q(t=0) = \frac{-\Gamma\omega_d V_0/L}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2} + A + B = q_0$$

$$I(t) = \frac{\Gamma\omega\omega_d V_0/L}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2}\sin(\omega_d t) + \frac{\omega_d V_0/L\,(\omega_0^2-\omega_d^2)}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2}\cos(\omega_d t)$$

$$-\left(\frac{\Gamma}{2}-\omega\right)Ae^{-\left(\frac{\Gamma}{2}-\omega\right)t} - \left(\frac{\Gamma}{2}+\omega\right)Be^{-\left(\frac{\Gamma}{2}+\omega\right)t}$$

$$I(t=0) = \frac{\omega_d V_0/L\,(\omega_0^2-\omega_d^2)}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2} - \left(\frac{\Gamma}{2}-\omega\right)A - \left(\frac{\Gamma}{2}+\omega\right)B = I_0$$

$$A = \frac{\Gamma\omega_d V_0/L}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2} + q_0 + B$$

$$\frac{\omega_d V_0/L\,(\omega_0^2-\omega_d^2) - \Gamma^2/2\cdot\omega_d V_0/L + \omega\Gamma\omega_d V_0/L}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2} - \left(\frac{\Gamma}{2}-\omega\right)q_0 \overbrace{- \left(\frac{\Gamma}{2}-\omega\right)B - \left(\frac{\Gamma}{2}+\omega\right)B}^{-\Gamma B} = I_0$$

$$\Rightarrow B = \frac{1}{\Gamma}\left\{\frac{(V_0/L)\omega_d\left[(\omega_0^2-\omega_d^2) - \Gamma^2/2 + \omega\Gamma\right]}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2} - \left(\frac{\Gamma}{2}-\omega\right)q_0 - I_0\right\}$$

$$\Rightarrow A = \frac{1}{\Gamma}\left\{\frac{(V_0/L)\omega_d\left[(\omega_0^2-\omega_d^2) + \omega\Gamma\right]}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2} + \left(\frac{\Gamma}{2}+\omega\right)q_0 - I_0\right\}$$

Critically Damped

$$q(t) = Re\left[\frac{-iV_0/L}{\omega_0^2-\omega_d^2+i\Gamma\omega_d}\, e^{i\omega_d t}\right] + Ae^{-\frac{\Gamma}{2}t} + Bte^{-\frac{\Gamma}{2}t}$$

$$= \frac{-\Gamma\omega_d V_0/L}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2}\cos(\omega_d t) + \frac{V_0/L\,(\omega_0^2-\omega_d^2)}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2}\sin(\omega_d t)$$

$$+ Ae^{-\frac{\Gamma}{2}t} + Bte^{-\frac{\Gamma}{2}t}$$

$$q(t=0) = \frac{-\Gamma\omega_d V_0/L}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2} + A = q_0$$

$$I(t) = \frac{\Gamma\omega\omega_d V_0/L}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2}\sin(\omega_d t) + \frac{\omega_d V_0/L\,(\omega_0^2-\omega_d^2)}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2}\cos(\omega_d t)$$

$$-\frac{\Gamma}{2}Ae^{-\frac{\Gamma}{2}t} + Be^{-\frac{\Gamma}{2}t} - \frac{\Gamma}{2}Bte^{-\frac{\Gamma}{2}t}$$

$$I(t=0) = \frac{\omega_d V_0/L\,(\omega_0^2-\omega_d^2)}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2} - \frac{\Gamma}{2}A + B = I_0$$

$$\Rightarrow A = \frac{\Gamma\omega_d V_0/L}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2} + q_0$$

$$\Rightarrow B = I_0 + \frac{\Gamma}{2}A - \frac{\omega_d V_0/L\,(\omega_0^2-\omega_d^2)}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2}$$

$$= I_0 + \frac{\Gamma}{2}q_0 + \frac{\omega_d V_0}{L}\cdot\frac{\Gamma^2/2-(\omega_0^2-\omega_d^2)}{(\omega_0^2-\omega_d^2)^2+(\Gamma\omega_d)^2}$$