Physics 15c (Hoffman)
Lecture #10
Thurs, Oct 7, 2010

Reading: H&L ch8 or Morin 7.2 (or a more advanced & detailed treatment of shock waves in Georgi chapter 14)

Doppler Effect & Shock Waves

Last time:

* average power carried by a mechanical wave: $\langle P \rangle = \frac{1}{2} Z \omega^2 \xi_0^2$

* impedance of a medium: $Z = \sqrt{E\rho_e}$

* dynamic range = ratio of largest detectable input to smallest detectable input

* human ear perceives wave intensity = power / area

* decibels = $10 \log\left(\frac{I}{I_0}\right)$ where $I_0 = 10^{-12}$ W/m² = minimum detectable intensity

* equation of state:
  $P \propto \frac{1}{V^\gamma}$ where $\gamma = \frac{\text{specific heat at constant } P}{\text{specific heat at constant } V} = \frac{C_P}{C_V} > 1$
  $\gamma = 5/3$ for monatomic (e.g. He); $\gamma = 7/5$ for diatomic (e.g. O_2, N_2)

* bulk modulus = $B = \frac{\text{stress}}{\text{strain}} = \frac{-\Delta P}{\Delta V/V} = \gamma P$

* $v_{sound} = \sqrt{\frac{B}{\rho}}$; $Z_0 = \sqrt{B\rho}$

* intensity $I = \frac{1}{2} Z_0 \omega^2 \xi_0^2 \Rightarrow \xi_0 = \frac{1}{\omega}\sqrt{\frac{2I}{Z_0}}$
  $\Delta p = \sqrt{2IZ_0}$

Goals for Today:

* Doppler effect
  * non-relativistic (e.g. sound)
  * relativistic (e.g. light)
* shock waves

Doppler Effect

So far we've talked about information & energy carried by a wave, but we assumed that the wave source is fixed.

What if the source of the waves moves?

CAREFUL! What is moving w.r.t. what?
Sound wave: there are actually 3 things that can move:
① source
② observer
③ medium (e.g. air) through which sound travels

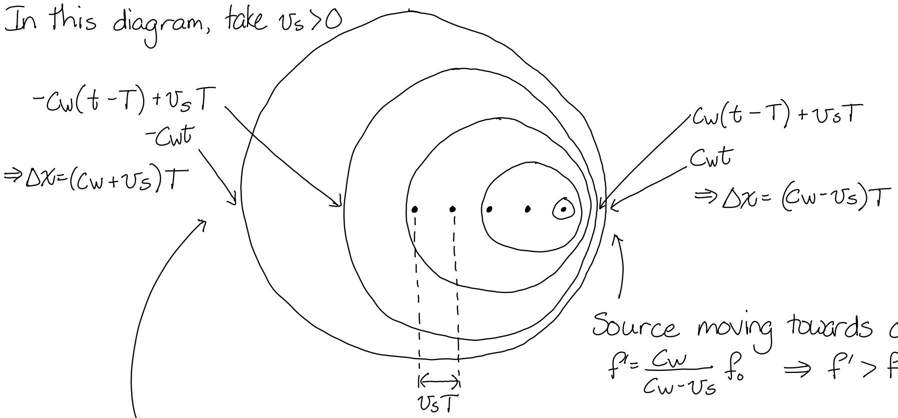
Moving source; stationary medium, observer:

c_w = speed of sound through medium
(c_w is always in the rest frame of the medium)
v_s = speed of moving source
(take $v_s > 0$ if source moves towards observer)
(assume $|v_s| < c_w$)
f_0 = original sound frequency

Follow 2 successive peaks of the sound wave:
→ they are generated $T = 1/f_0$ apart in time
→ source moves $v_s T = v_s/f_0$ during this time
→ at time $t=0$:
① 1st peak is at $x_0 = 0$
② 2nd peak doesn't yet exist
→ at time $t = T$:
① 1st peak is at $x_1 = c_w T = c_w/f_0$
② 2nd peak is at $x_2 = v_s T = v_s/f_0$
$\Rightarrow \Delta x = (c_w - v_s)/f_0$
→ now this spacing between peaks Δx will continue towards observer at speed c_w
→ so observer perceives the arrival of the two peaks separated by a time
$T' = \Delta x/c_w = [(c_w - v_s)/c_w]/f_0 = T(c_w - v_s)/c_w$

$\boxed{f' = \frac{c_w}{c_w - v_s} f_0}$ Doppler shift for moving source, stationary medium, stationary observer, $v_s < c_w$

In this diagram, take $v_s > 0$



Source moving towards observer:

$f' = \frac{c_w}{c_w - v_s} f_0 \Rightarrow f' > f_0$

Source moving away from observer:

$f' = \frac{c_w}{c_w + v_s} f_0 \Rightarrow f' < f_0$

Combine: $\boxed{f' = \frac{c_w}{c_w \mp v_s} f_0}$ source moving with respect to stationary observer (− for towards; + for away)

Stationary source, medium; moving observer:

c_w = speed of sound in stationary medium
v_0 = speed of observer (toward source)
f_0 = original sound frequency

Follow 2 peaks of the sound waves:

→ they are generated $T = 1/f_0$ apart
→ the spacing between peaks is $\lambda = c_wT = c_w/f_0$
→ at time $t=0$
  ① suppose observer hears first peak
  ② 2nd peak is c_w/f_0 away

2nd peak $\xrightarrow{}$ c_w $\xleftarrow{}$ v_0 observer, just as she hears 1st peak

$\underbrace{c_w \quad v_0}_{\lambda = c_w/f_0}$

→ 2nd peak meets observer at time $T' = \frac{c_w/f_0}{c_w + v_0}$

$\boxed{f' = \frac{c_w + v_0}{c_w} f_0}$ observer moving towards stationary source (stationary medium)

What if the observer moves away from the source?

2nd peak $\xrightarrow{}$ c_w $\xrightarrow{}$ v_0 observer, just as she hears 1st peak

$\underbrace{c_w}_{\lambda = c_w/f_0}$

still taking $v_0 > 0$ to be a speed, not velocity

→ 2nd peak meets observer at time $T' = \frac{c_w/f_0}{c_w - v_0}$

$\Rightarrow f' = \frac{c_w - v_0}{c_w} f_0$

Combine: $\boxed{f' = \frac{c_w \mp v_0}{c_w} f_0}$ observer moving with respect to stationary source (− for away, + for towards)

Moving source & observer; stationary medium:

Combine: $f' = \frac{c_w \mp v_0}{c_w \mp v_s} f_0$

Fold the directions of motion into the signs of v_0, v_s so they become velocities instead of speeds:

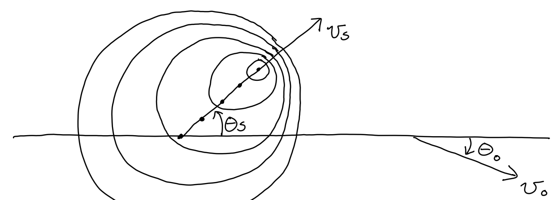
$v_0 < 0$ for motion towards source
$v_0 > 0$ for motion away from source
$v_s > 0$ for motion towards observer
$v_s < 0$ for motion away from observer

In summary, define the signs of v_0, v_s with respect to the direction of sound propagation, c_w.

Non-parallel velocities:

What if v_0, v_s not parallel to the sound c_w?

→ just take the component that is parallel to the sound



$\boxed{f' = \frac{c_w - v_0 \cos\theta_0}{c_w - v_s \cos\theta_s}}$

What about relativity?
Why does it matter who (source, observer) is moving?

* Because <u>medium</u> (e.g. air) gives an absolute reference frame for sound.
  Sound cannot travel without a medium!

We measure v_o and v_s w.r.t. medium
If there is wind (motion of medium) → must add in v_m

Moving source; moving medium, moving observer (away)
→ two ways to think of this

① speed of sound is $c_w + v_m$ in rest frame of Earth

$$\Rightarrow f' = \frac{(c_w+v_m)-v_o}{(c_w+v_m)-v_s} f_o$$

② speed of sound is c_w in rest frame of medium
observer speed is $v_o - v_m$ in rest frame of medium
source speed is $v_s - v_m$ in rest frame of medium

$$\Rightarrow f' = \frac{c_w-(v_o-v_m)}{c_w-(v_s-v_m)} f_o = \frac{c_w+v_m-v_o}{c_w+v_m-v_s} f_o$$

What if any of these 3 (observer, source, medium) are moving at relativistic speeds with respect to each other?
→ velocities add relativistically:

$$v_{tot} = \frac{v_1+v_2}{1+v_1v_2/c^2}$$

→ need to consider time dilation:

$$t' = \gamma t \quad \text{where} \quad \gamma = \sqrt{\frac{1}{1-v^2/c^2}}$$

... not very realistic ... can't really think of examples of mechanical waves moving at relativistic speeds, so we won't pursue this in detail ...

Electromagnetic waves

no medium → no absolute reference frame!
→ whether source or observer is moving shouldn't matter

Suppose source & observer are approaching each other
→ we should be able to consider it in 2 ways

moving source $\longrightarrow v$ · stationary observer	stationary source · $v \longleftarrow$ moving observer
$f' = \frac{c}{c-v} f_o$	$f' = \frac{c+v}{c} f_o$

These 2 formulas are not the same!
They should be!
Light must follow a different rule!

→ need to consider time dilation!
→ clock runs fastest in its own reference frame (See Morin pages 514-515)
→ any reference frame moving w.r.t. clock sees time running slower by a factor: $\gamma = \sqrt{\frac{1}{1-v^2/c^2}}$

① Source moves at v, observer stationary
→ $T = 1/f_o$ = time between peaks in source frame
→ $T' = \gamma T = \gamma/f_o$ = time between peaks in stationary frame of observer
→ but source moving towards observer also gives us the regular Doppler effect:

$$\Rightarrow f' = \frac{c}{c-v}\frac{f_o}{\gamma} = \frac{1}{1-v/c}\sqrt{\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right)}\, f_o$$

$$= \sqrt{\frac{1+v/c}{1-v/c}}\, f_o = \sqrt{\frac{c+v}{c-v}}\, f_o \quad \text{(moving towards!)}$$

② Source stationary, observer moves at v
→ $T = 1/f_o$ = time between peaks in source frame
→ $T' = T/\gamma = 1/(\gamma f_o)$ = time between peaks in observer frame
→ but observer moving towards source also gives us the regular Doppler effect:

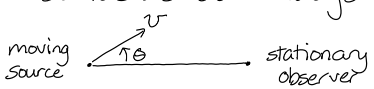
$$\Rightarrow f' = \frac{c+v}{c}\gamma f_o = \left(1+\frac{v}{c}\right)\sqrt{\frac{1}{\left(1+\frac{v}{c}\right)\left(1-\frac{v}{c}\right)}}\, f_o$$

$$= \sqrt{\frac{1+v/c}{1-v/c}}\, f_o = \sqrt{\frac{c+v}{c-v}}\, f_o \quad \text{(moving towards!)}$$

EQUAL!

Does time dilation really matter for other waves?
- → only if v is very large, close to c
- → never an issue with sound
  (or any other medium waves I can think of...)

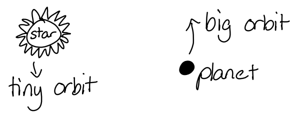
Remember that time dilation doesn't depend on direction of v
- → so factors don't always cancel so nicely if things move at angles



$$f = \underbrace{\sqrt{1-\frac{v^2}{c^2}}}_{\text{time dilation}} \; \underbrace{\frac{c}{c - v\cos\theta}}_{\text{moving source at angle } \theta} f_0$$

Example: planet detection

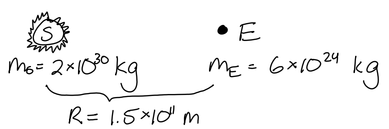
Stars are luminous → we can see them directly



star will get slightly closer/farther
→ can we detect change in brightness?

order of magnitude estimate:

total luminosity of sun = 4×10^{26} W
wobble of sun due to Earth



CM is $\frac{m_E}{m_S} = 3\times10^{-6}$ of the way from S to E
$= 4.5\times10^{5}$ m from sun

suppose we are 100 light years away
$=(100\text{ yrs})(\pi\times10^7 \text{ s/yr})(3\times10^8 \text{ m/s}) = \cancel{4}10^{18}$ m

intensity falls off like $1/r^2$

$$\Rightarrow \frac{\Delta I}{I} = \frac{\frac{d}{dr}\left(\frac{1}{r^2}\right)\Delta r}{\frac{1}{r^2}} = \frac{\frac{-2}{r^3}\Delta r}{\frac{1}{r^2}} = -\frac{2\Delta r}{r} \approx \frac{10^6 \text{ m}}{10^{18}\text{ m}} = 10^{-12}$$

How many photons is this?

telescope lens 1m diameter

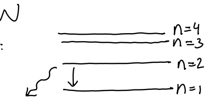
total fraction of star power captured by telescope

$$\sim \frac{(\frac{1}{2}m)^2\pi}{4\pi(10^{18}m)^2} = \frac{1}{16}\times10^{-36} \approx 6\times10^{-38}$$

total star power captured by telescope

$$\sim (6\times10^{-38})(4\times10^{26}\text{ W}) = 2\times10^{-11}\text{ W}$$

energy of Lyman α-line in hydrogen:



$\lambda = 1216$ Å

$f = c/\lambda = (3\times10^8 \text{ m/s})/(1.2\times10^{-7}\text{ m})$
$= 3\times10^{15}$ Hz

energy $= hf = (6.6\times10^{-34}\text{ J·s})(3\times10^{15}\text{ Hz}) = 2\times10^{-18}$ J

if <u>all</u> star's energy were at this f (<u>it's not</u>!)
then we would have

$$\frac{2\times10^{-11}\text{ W}}{2\times10^{-18}\text{ J}} = 10^7 \text{ photons/second from star}$$

we'd be trying to detect 1 part in 10^{12} of these
≈ 1 extra photon every 10^5 seconds

$$10^5 \text{ s} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{1\text{ hr}}{60\text{ min}} \times \frac{1\text{ day}}{24\text{ hrs}} = 1.5\text{ days}$$

<u>NOT LIKELY</u>!

⇒ Use Doppler shift instead
frequencies are <u>much</u> easier to measure than intensities

QUIZ: what's the fractional Doppler shift in this case?

What's v of star?

$$v = \frac{2\pi(4.5\times10^5\,m)}{\pi\times10^7\,s} = 0.1\ m/s$$

$$f' = \underbrace{\sqrt{\frac{c+v}{c-v}}}\,f_0$$

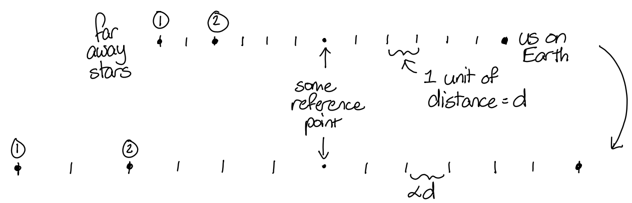
$$\left(1+\frac{2v}{c-v}\right)^{1/2} \approx \left(1+\frac{2v}{c}\right)^{1/2} \approx 1+\frac{v}{c}$$

$$\frac{v}{c} = \frac{0.1\ m/s}{3\times10^8\ m/s} = 3\times10^{-10}$$

still not easy, but within the realm of possibility!
→ detect frequencies via beats
(mix with nearby reference frequency
→ need very accurate reference frequency)

Example: redshift of our universe

space itself is expanding



After time t, space has expanded by some factor α.

① has moved away at $v_1 = \dfrac{(12d)\alpha - 12d}{t} = \dfrac{12d(\alpha-1)}{t}$

② has moved away at $v_2 = \dfrac{(10d)\alpha - 10d}{t} = \dfrac{10d(\alpha-1)}{t}$

⇒ velocity at which the star recedes is proportional to the distance from the star!

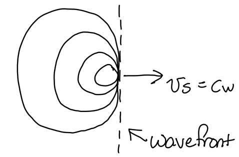
⇒ can use redshift (lowered frequency) of light from star to tell its distance!

ALSO, the farther away the star is, the longer it took for light to get here. So the greater the redshift, the farther back in time we are looking!

Shock Waves

What happens when a source moves at (or greater than) c_w?

waves pile up → SHOCK WAVE

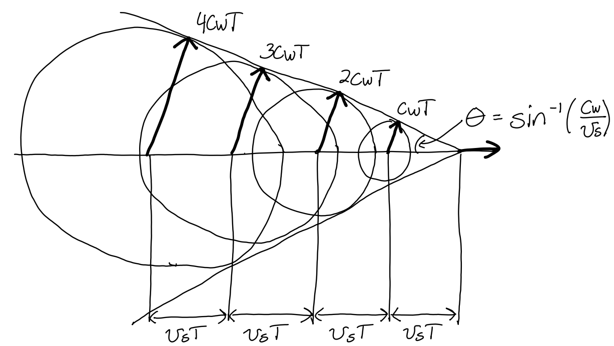


theoretically, the energy pile-up at the wavefront is ∞
→ what actually happens?
very complicated fluid dynamics problem

What useful, simple facts can we extract from this?

→ turbulence from big energy concentration → HUGE drag
⇒ very hard to break the sound barrier
(not a physics impossibility like c, just an engineering challenge)
→ once we exceed the speed of sound, the shock wave becomes a cone

MACH cone:



→ energy concentrates at the cone surface
→ where the cone touches the observer, the observer hears "sonic boom" = rapid delivery of LOTS of energy

Summary

Doppler shift (non-relativistic):

$$f' = \frac{(c_w + v_m) - v_o}{(c_w + v_m) - v_s} f_o$$

Doppler shift (relativistic):

$$f' = \sqrt{\frac{c+v}{c-v}}\, f_o$$

Mach cone angle: $\theta = \sin^{-1}\left(\frac{c_w}{v_s}\right)$

Next time:

Standing waves & musical instruments

Reading for next time: H & L, chapter 6
or Morin chapter 4
or Georgi section 5.3