Can you hear the shape of a drum?

Can you hear the shape of a drum?

1964 – Milnor proves the existence of two noncongruent 16-dimensional flat tori in which the Laplacian's eigenvalue spectrum is identical

1966 – Kac asks the more general question: “can you hear the shape of a drum?”,

In later decades, mathematicians struggled to find such isospectral geometries in lower-dimensional systems.

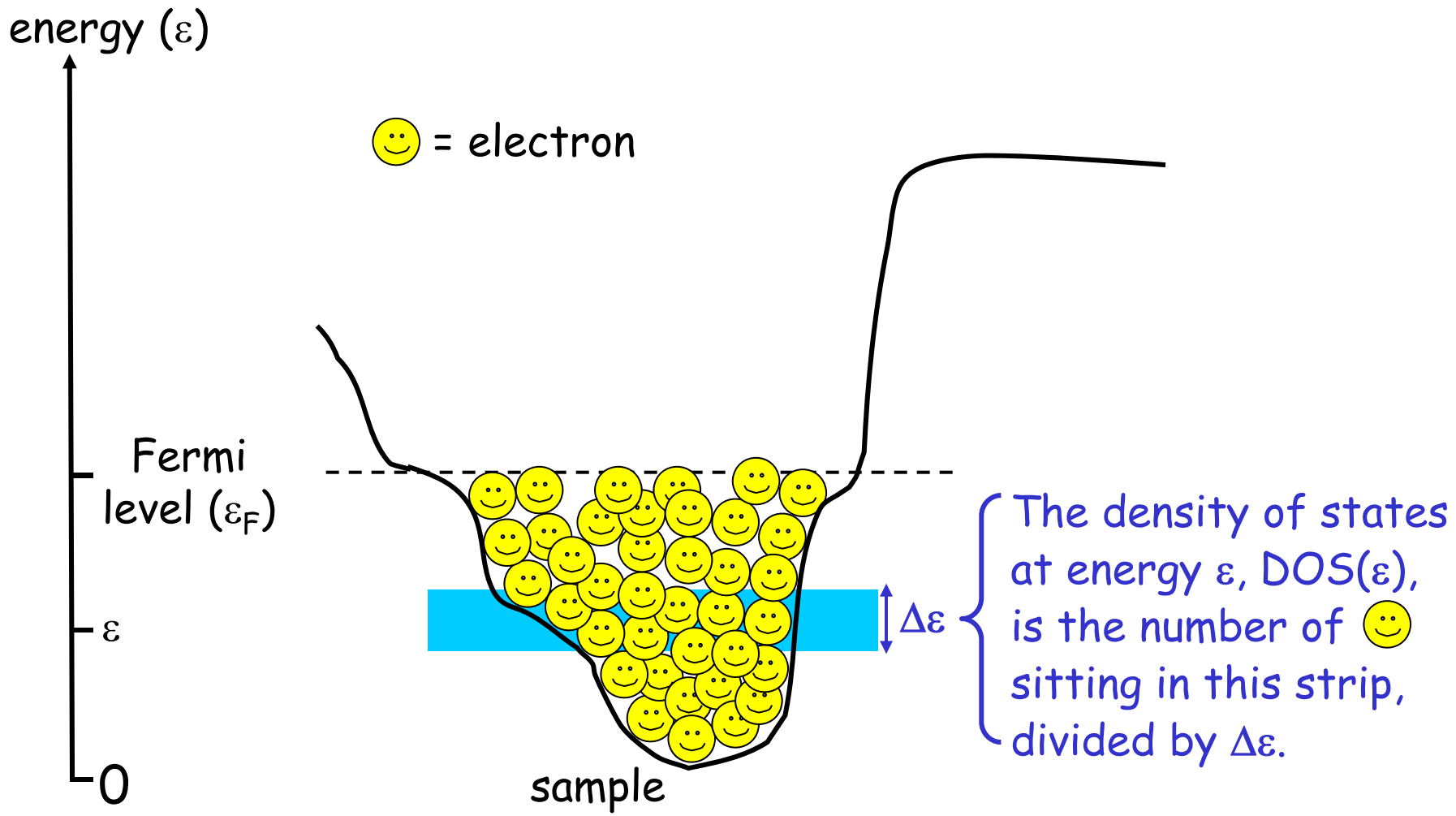
1980s – Sunada discovered a group-theoretic method for proving isospectrality.
Buser proved a class of isospectral manifolds by "pasting" two-dimensional (2D) flat tiles together in higher dimensions.

However, the dimensionality of proven isospectral domains was only slowly whittled down to three dimensions.

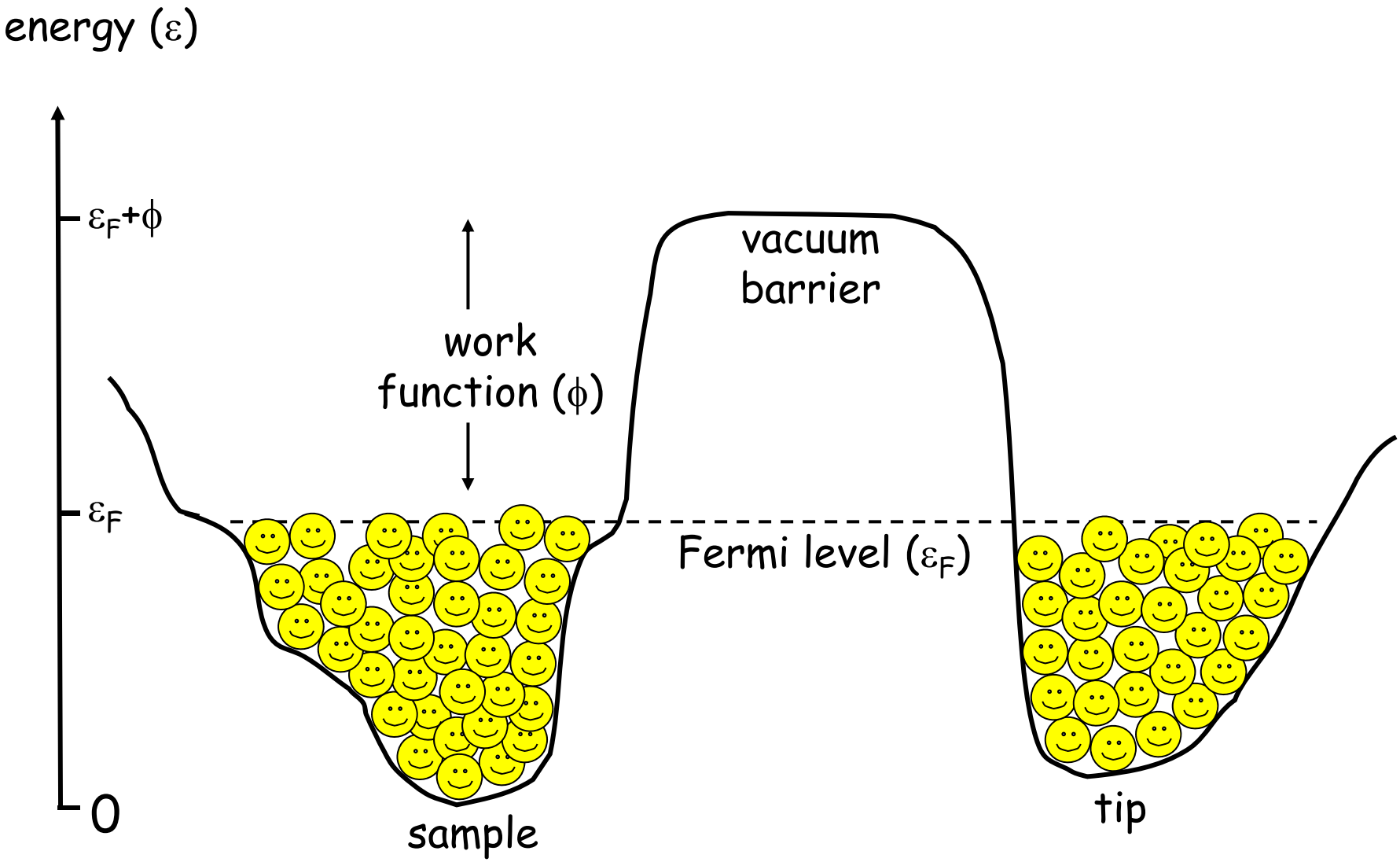
What about one dimension?

1992 – Gordon *et al*. mathematically discovered the first 2D isospectral domains finally answering Kac's enigma.

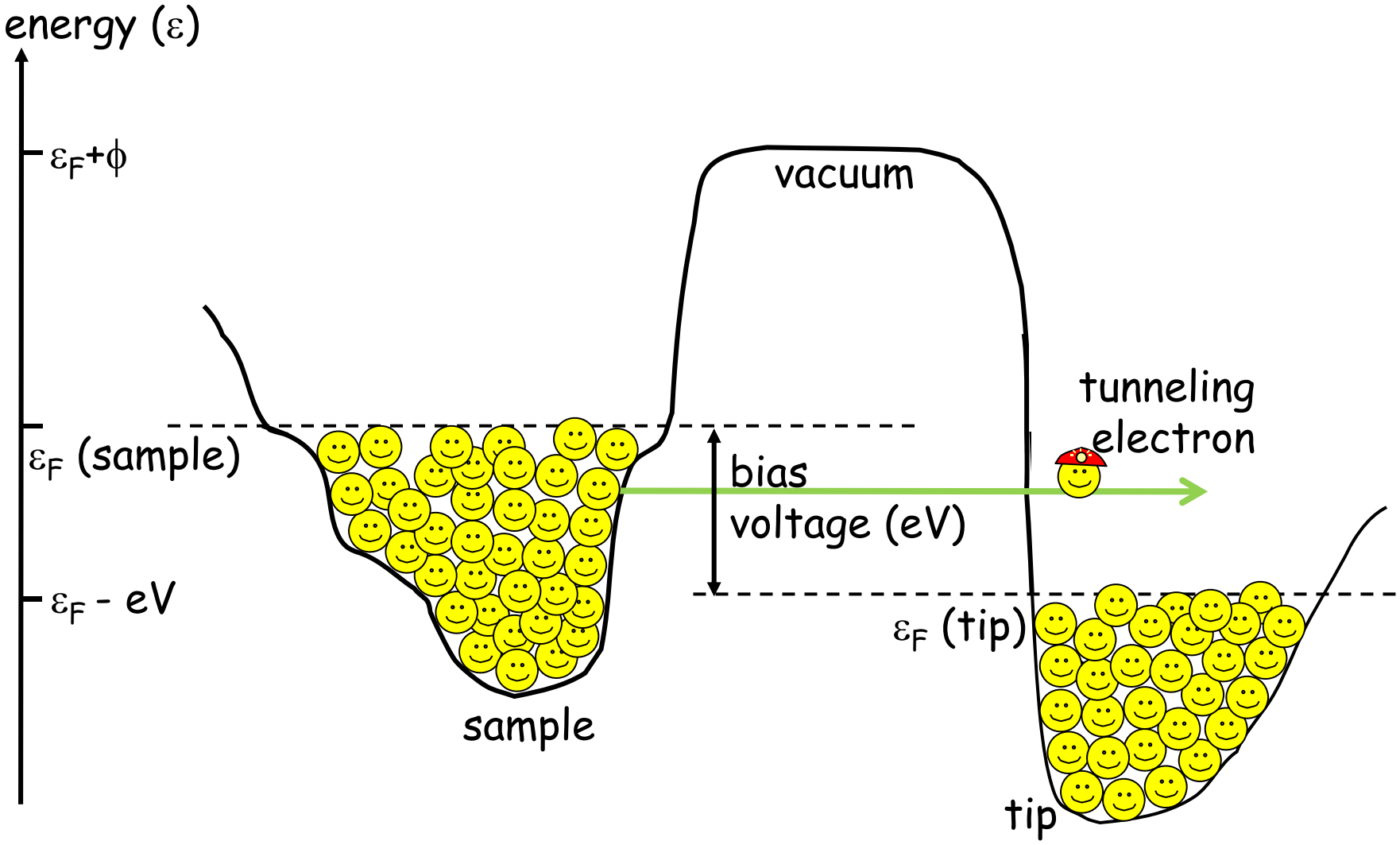
Scanning Tunneling Microscopy



Scanning Tunneling Microscopy

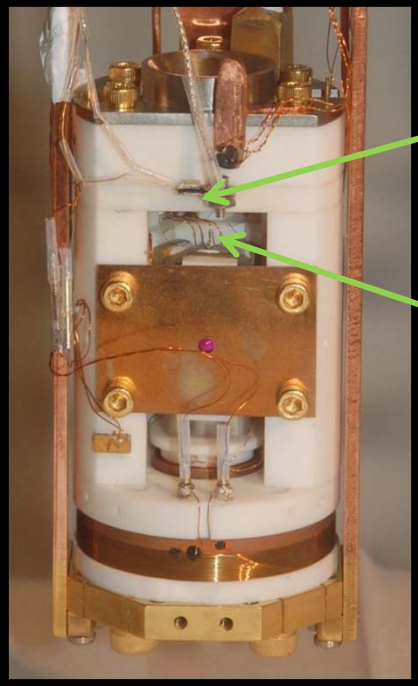


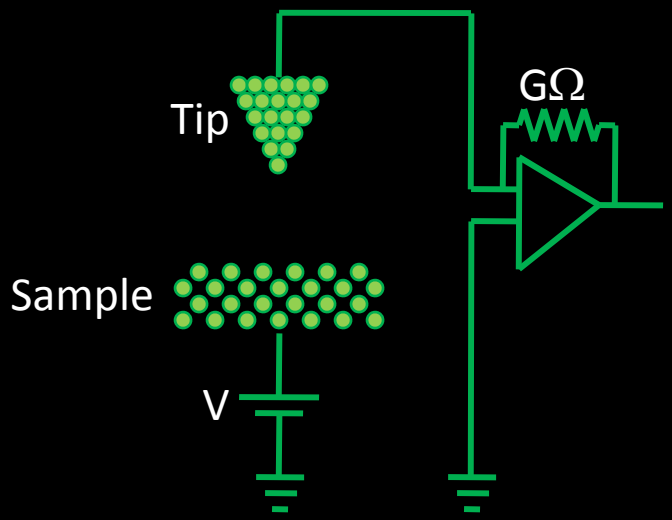
Scanning Tunneling Microscopy

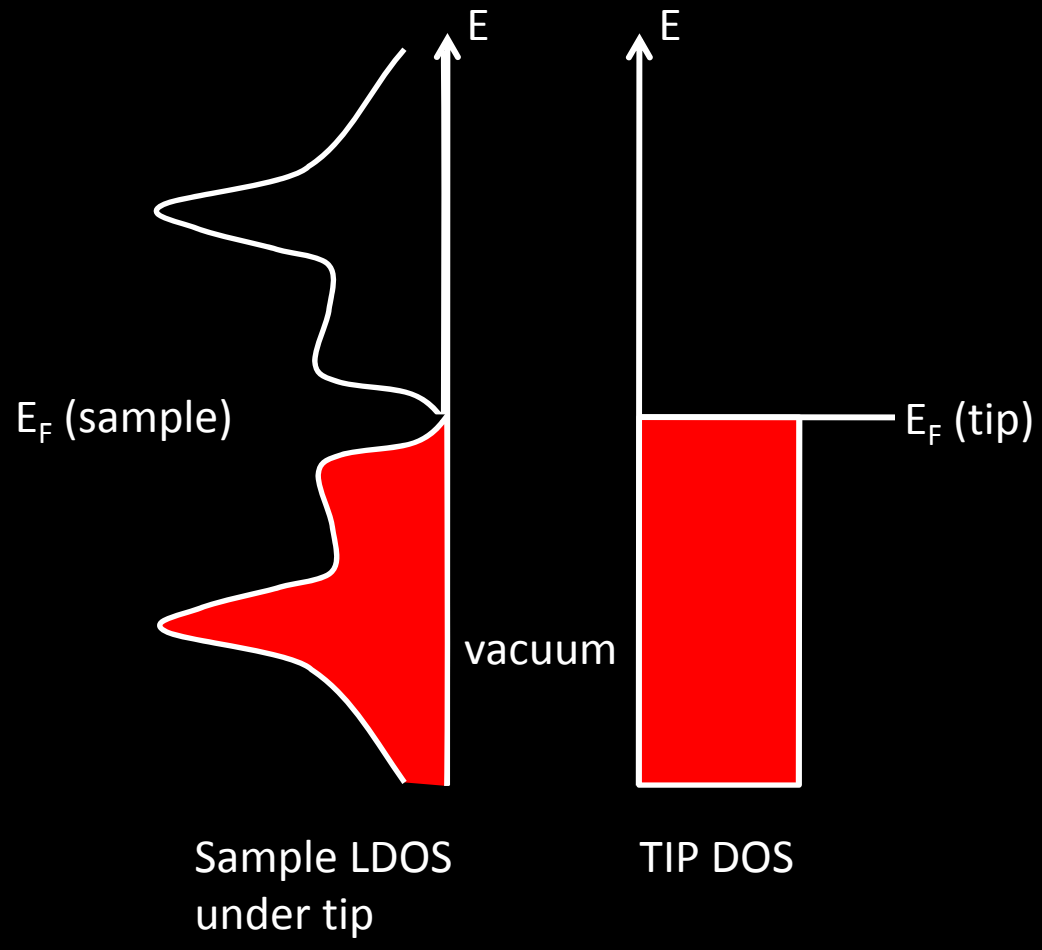


$I \sim$ (integrated density of states) $\times e^{-2\kappa d}$

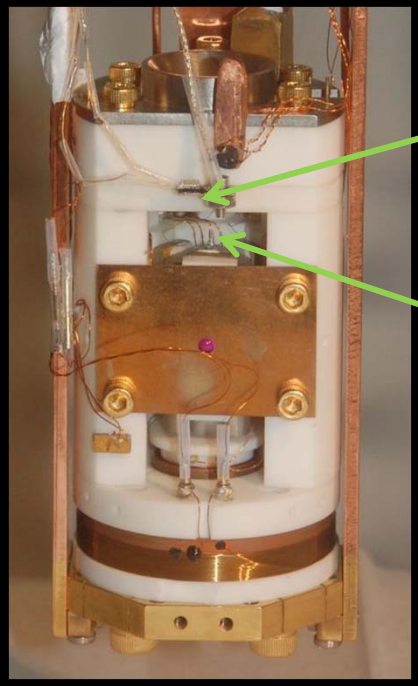
A little more professional introduction to STM…

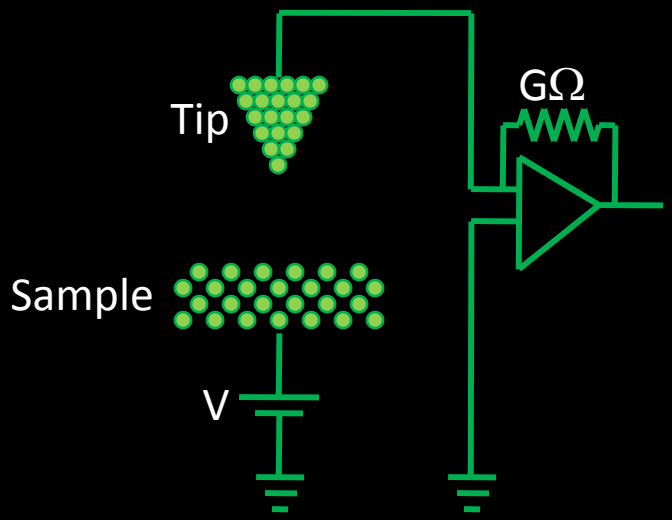




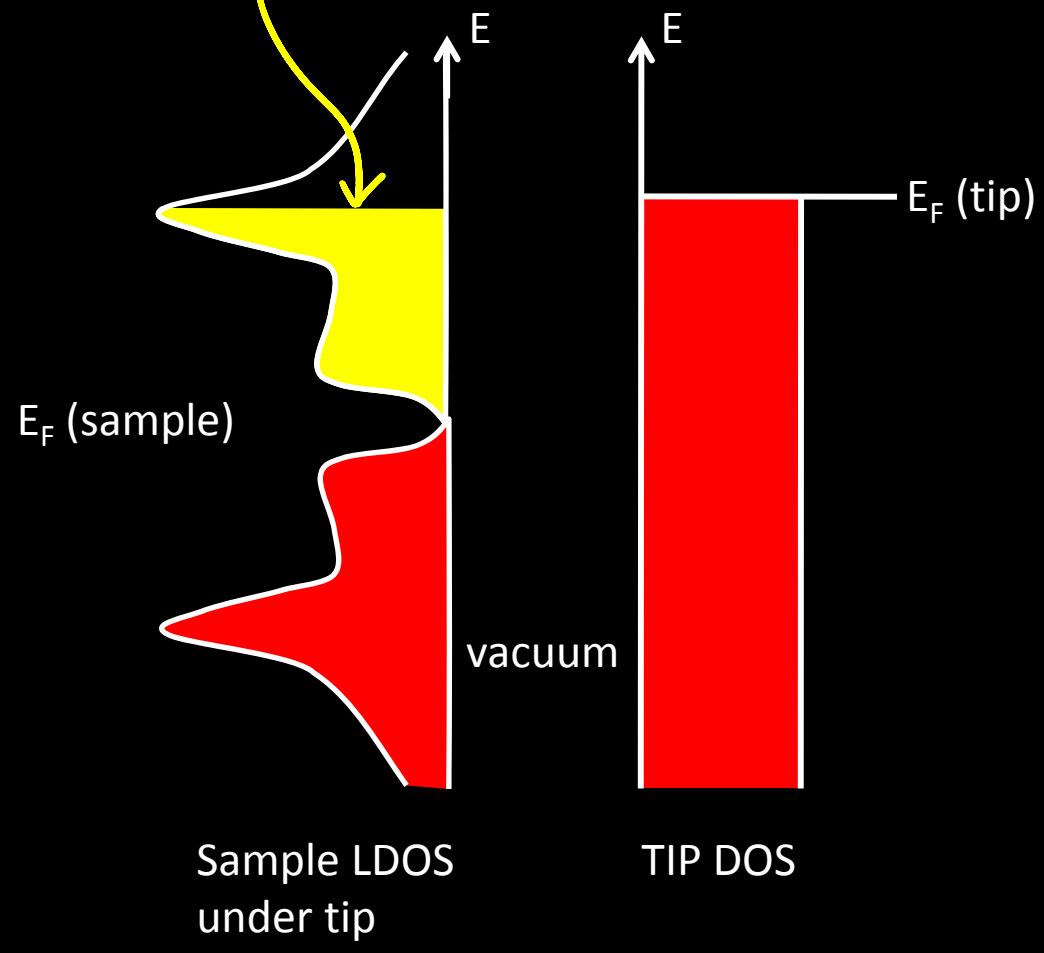


A little more professional introduction to STM…





$$I(V) = \int_{E_F}^{eV} LDOS(E)\,dE$$



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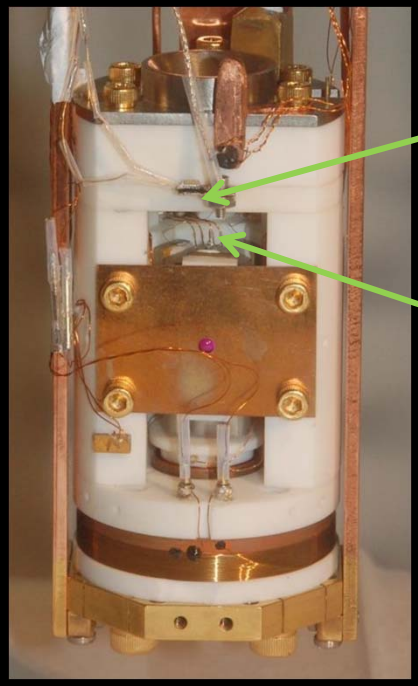
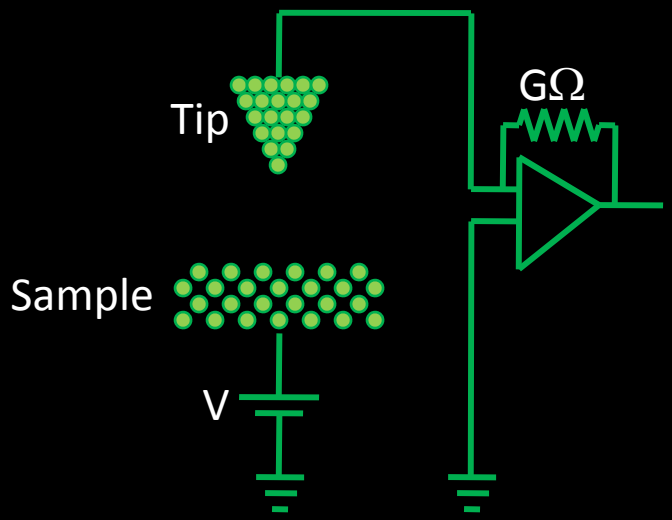
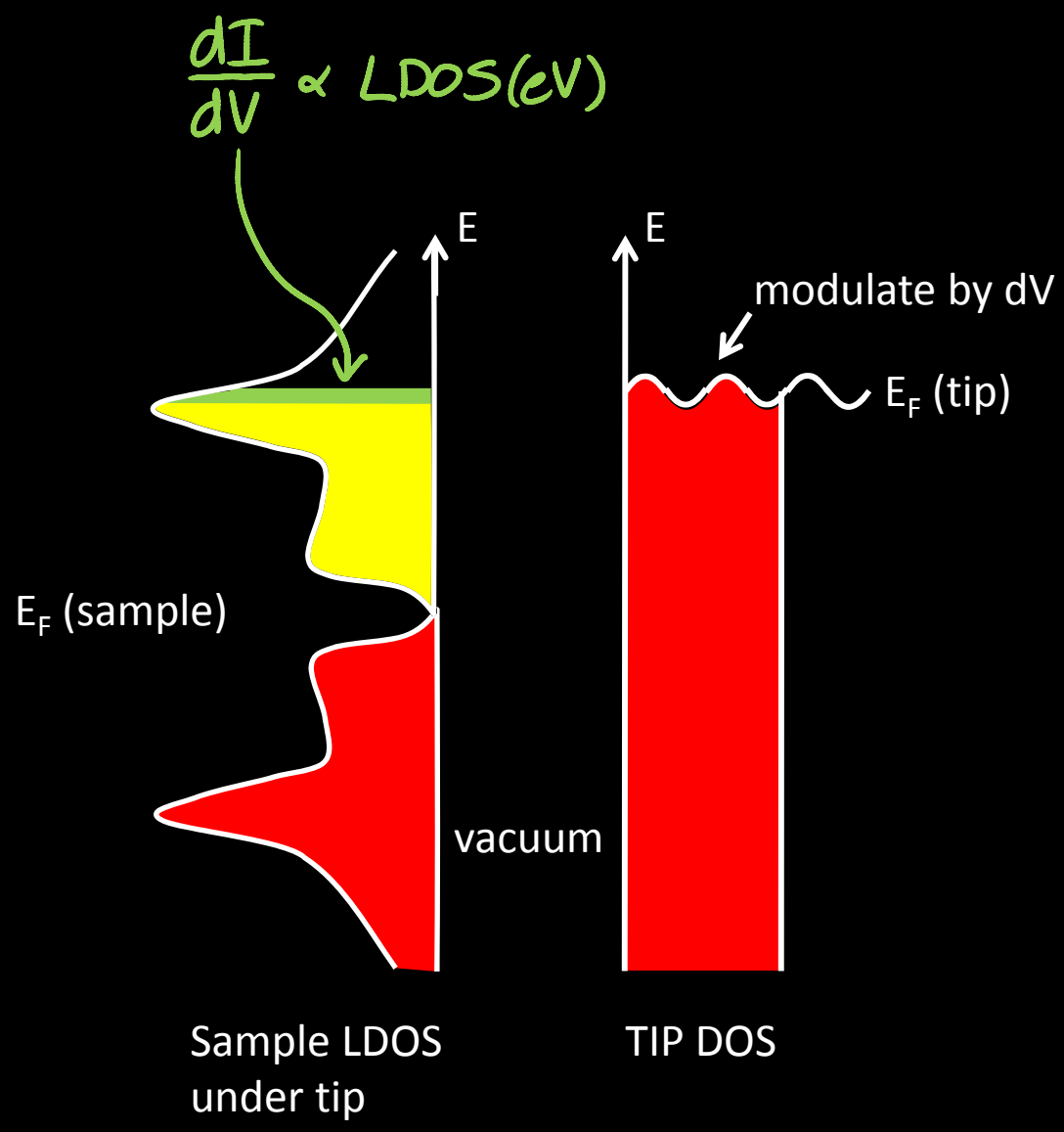
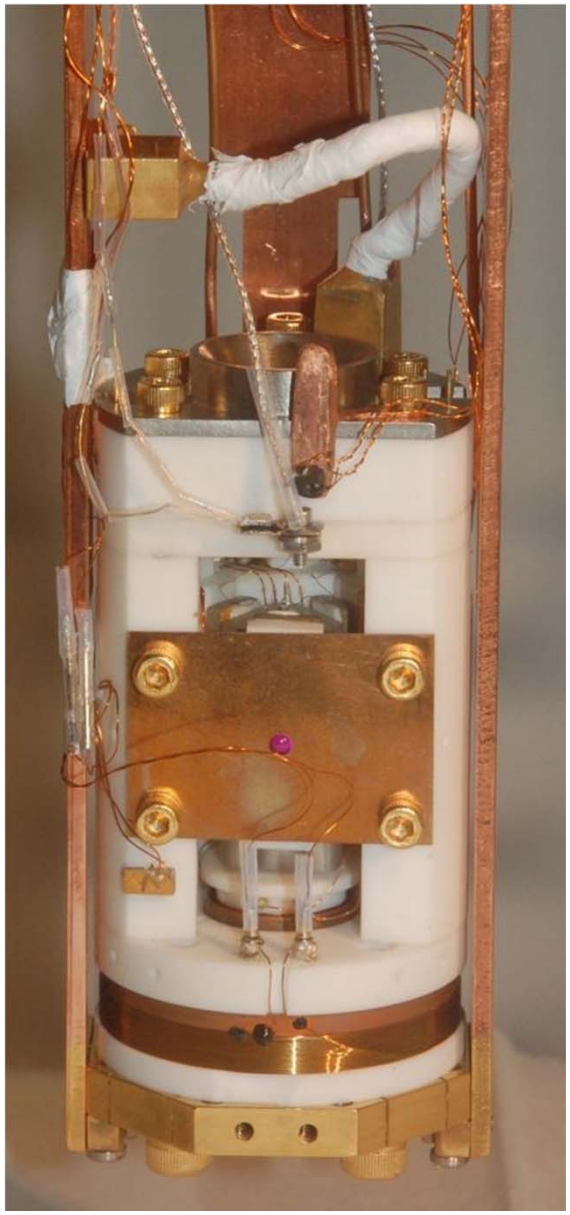
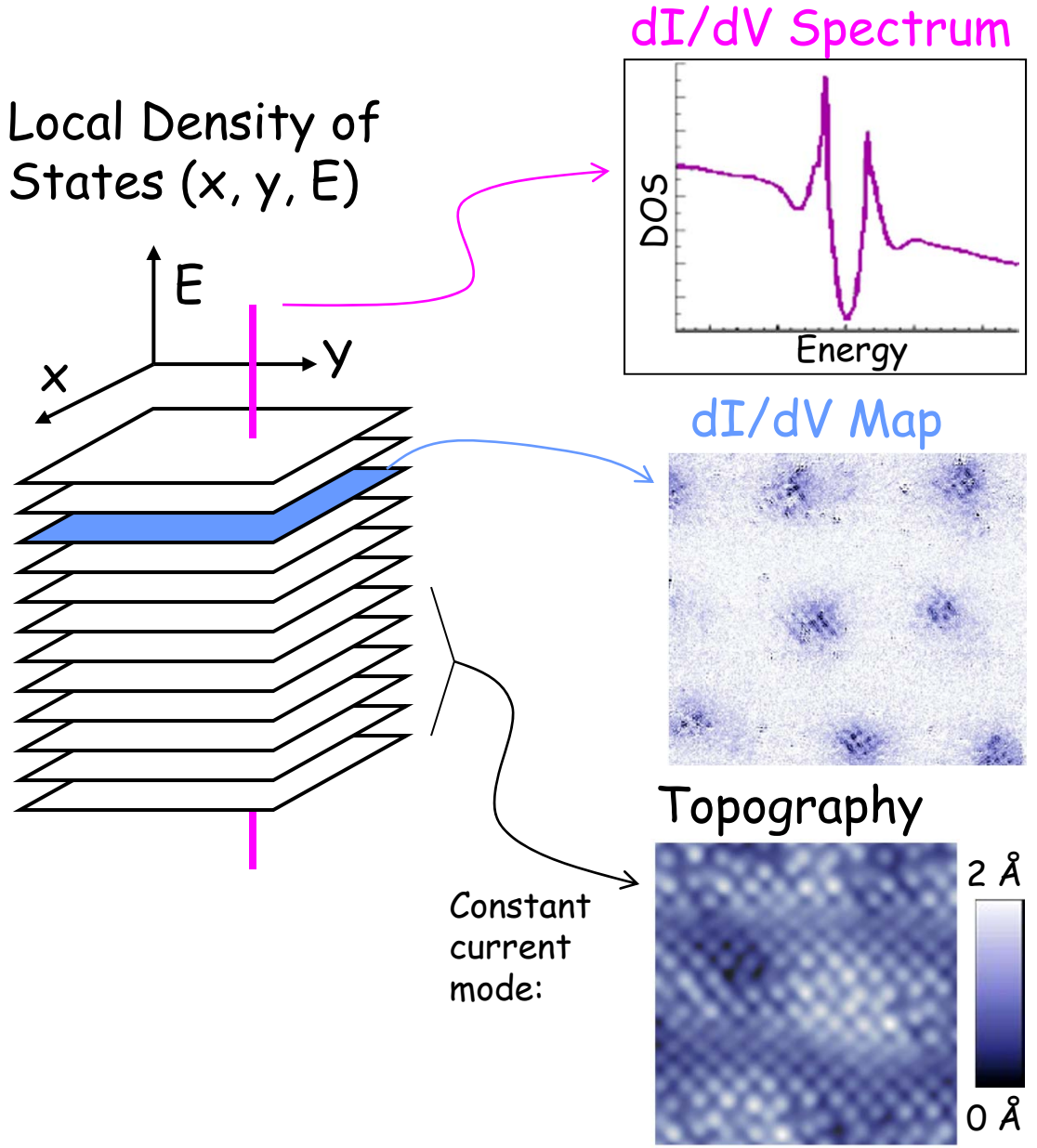


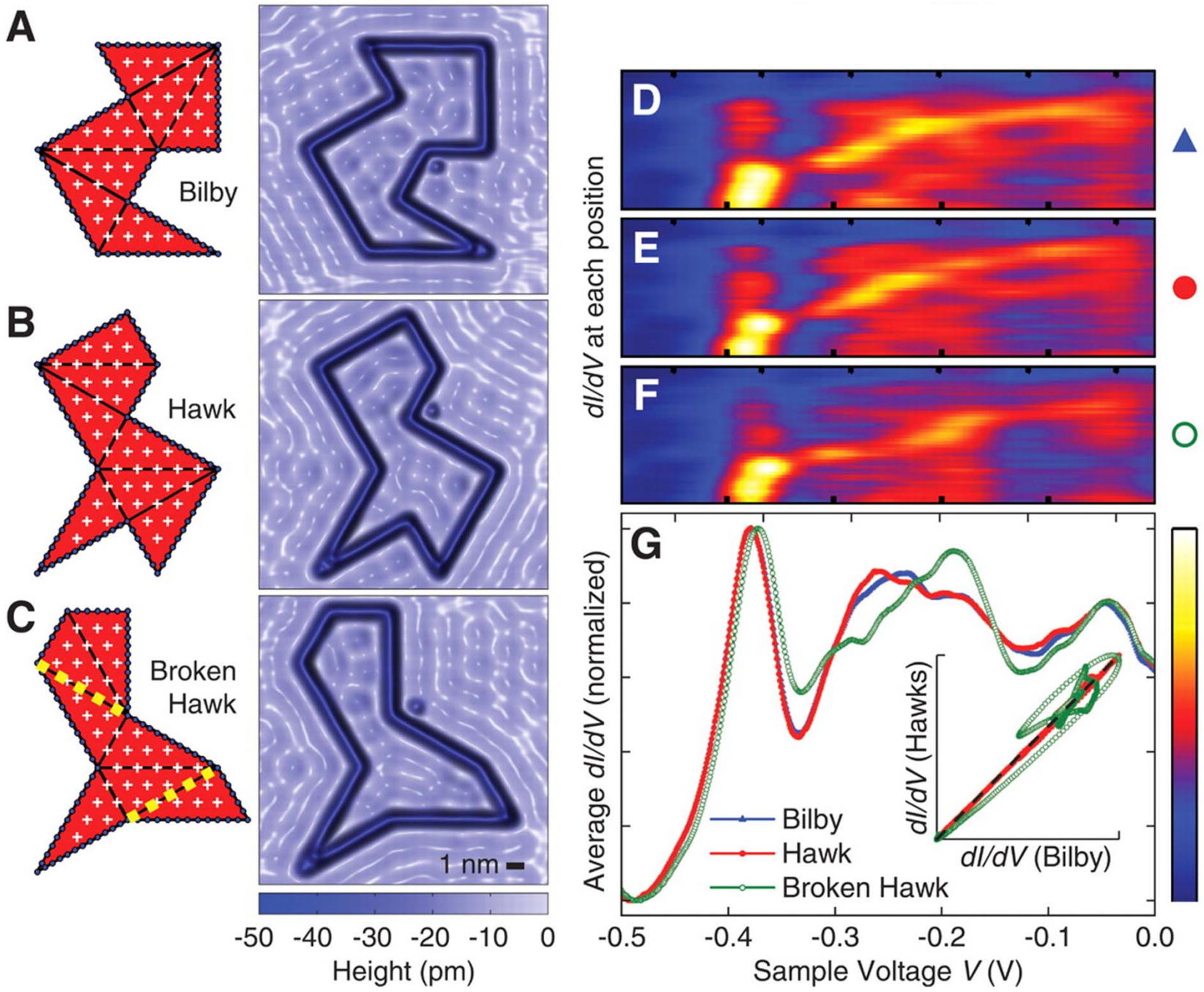




Types of STM Measurement





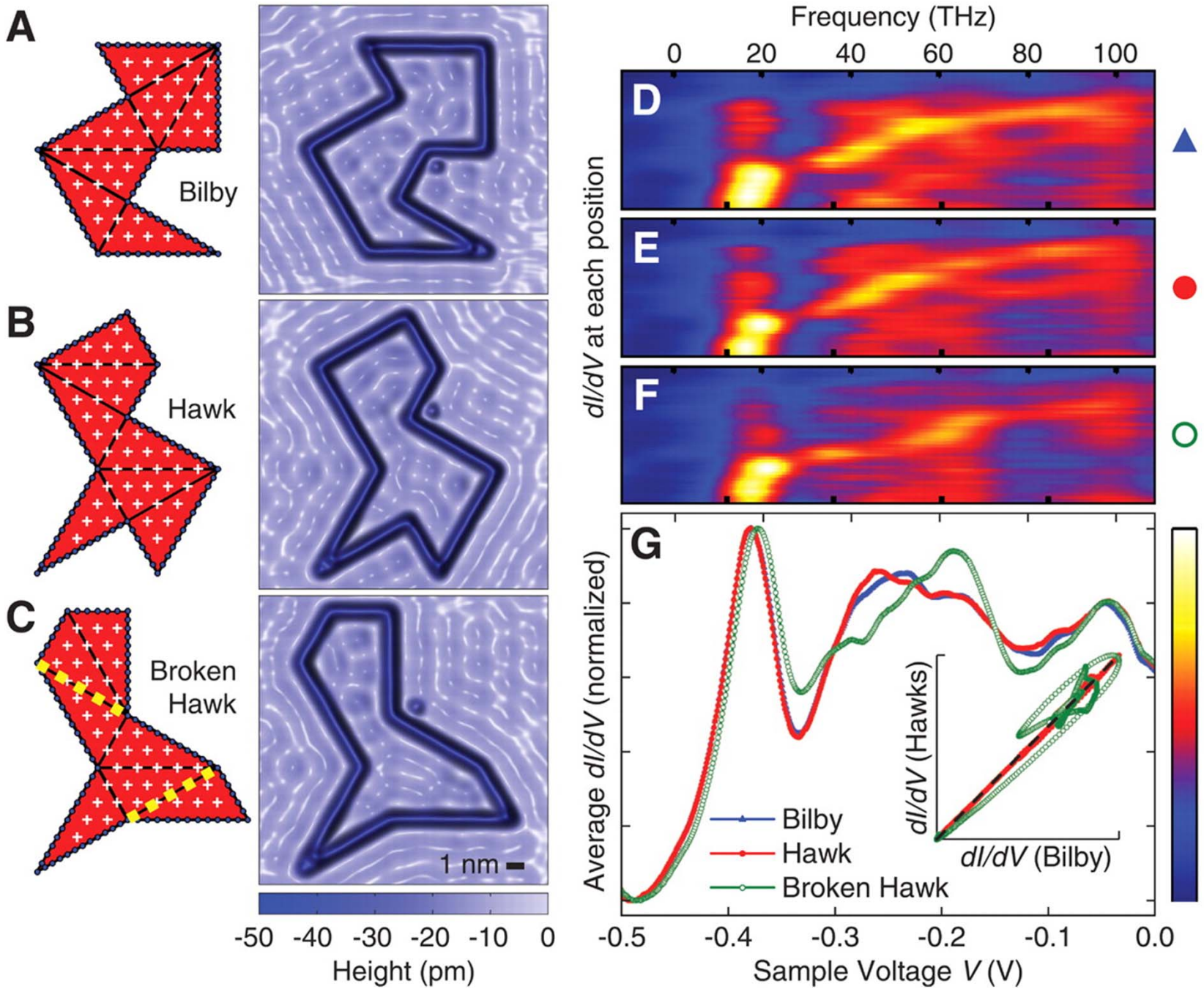

A
Bilby
B
Hawk
C
Broken
Hawk
1 nm
-50 -40 -30 -20 -10 0
Height (pm)
D
E
F
dI/dV at each position
G
Average dI/dV (normalized)
Bilby
Hawk
Broken Hawk
dI/dV (Hawks)
dI/dV (Bilby)
-0.5 -0.4 -0.3 -0.2 -0.1 0.0
Sample Voltage V (V)

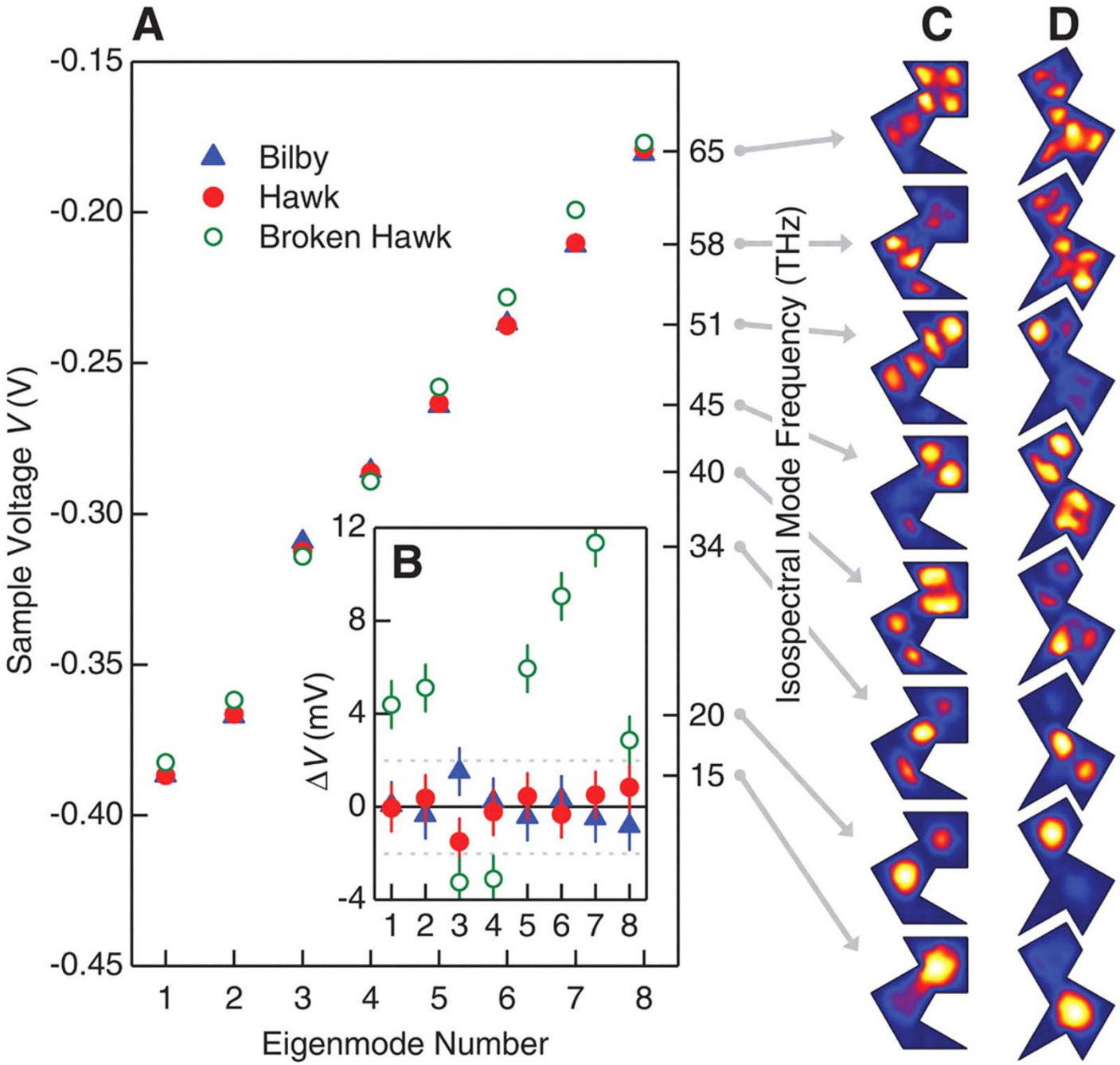
Size of structure: L ~10 nm
Fermi velocity: $v_F \sim 10^8$ cm/s
What are the approximate frequencies of the electron waves?

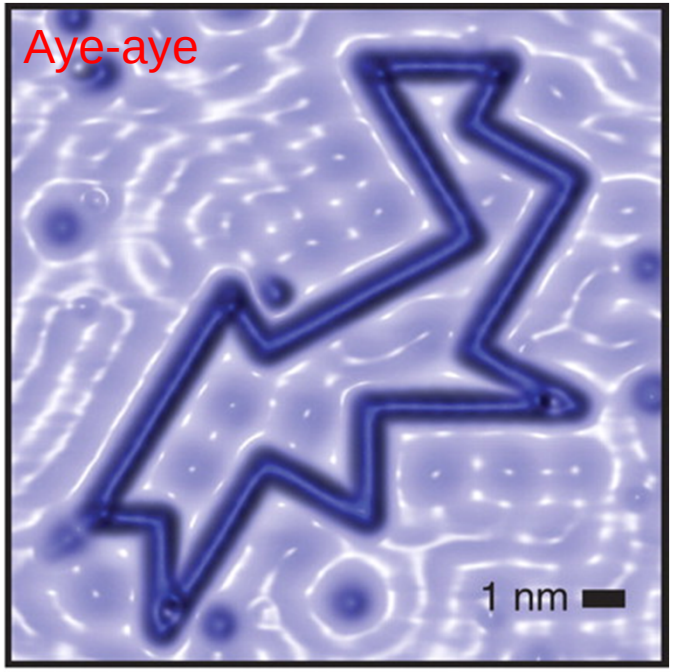
$\lambda/2 \sim 10^{-8}$ m
$v_F \sim 10^6$ m/s

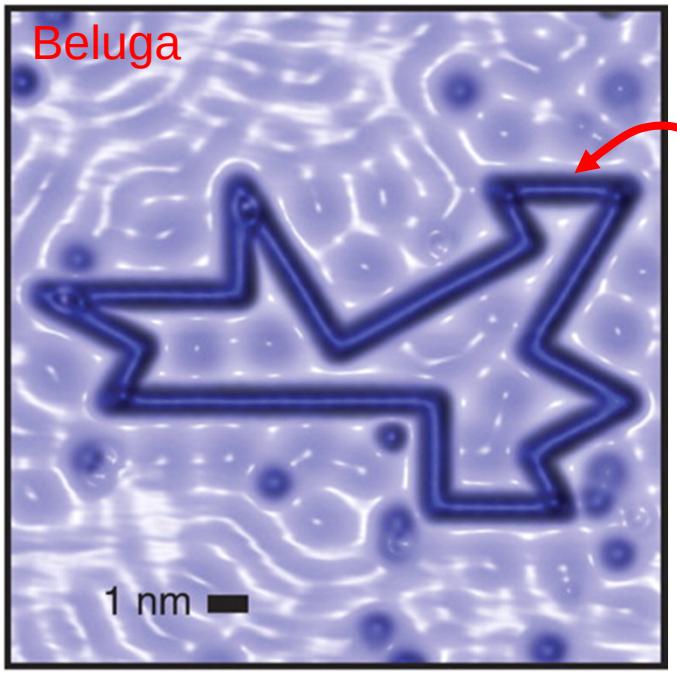
$f = v_F/\lambda = (10^6 \text{ m/s}) / (10^{-8} \text{ m}) = 10^{14} \text{ Hz} = 100$ TeraHertz

$E = hf = (6.6 \times 10^{-34} \text{ J·s}) (10^{14} \text{ s}^{-1}) / (1.6 \times 10^{-19} \text{ J/eV}) = 0.41$ eV


A
Bilby
B
Hawk
C
Broken Hawk
1 nm
-50 -40 -30 -20 -10 0
Height (pm)
Frequency (THz)
0 20 40 60 80 100
D
E
F
dI/dV at each position
G
Average dI/dV (normalized)
Bilby
Hawk
Broken Hawk
dI/dV (Hawks)
dI/dV (Bilby)
-0.5 -0.4 -0.3 -0.2 -0.1 0.0
Sample Voltage V (V)

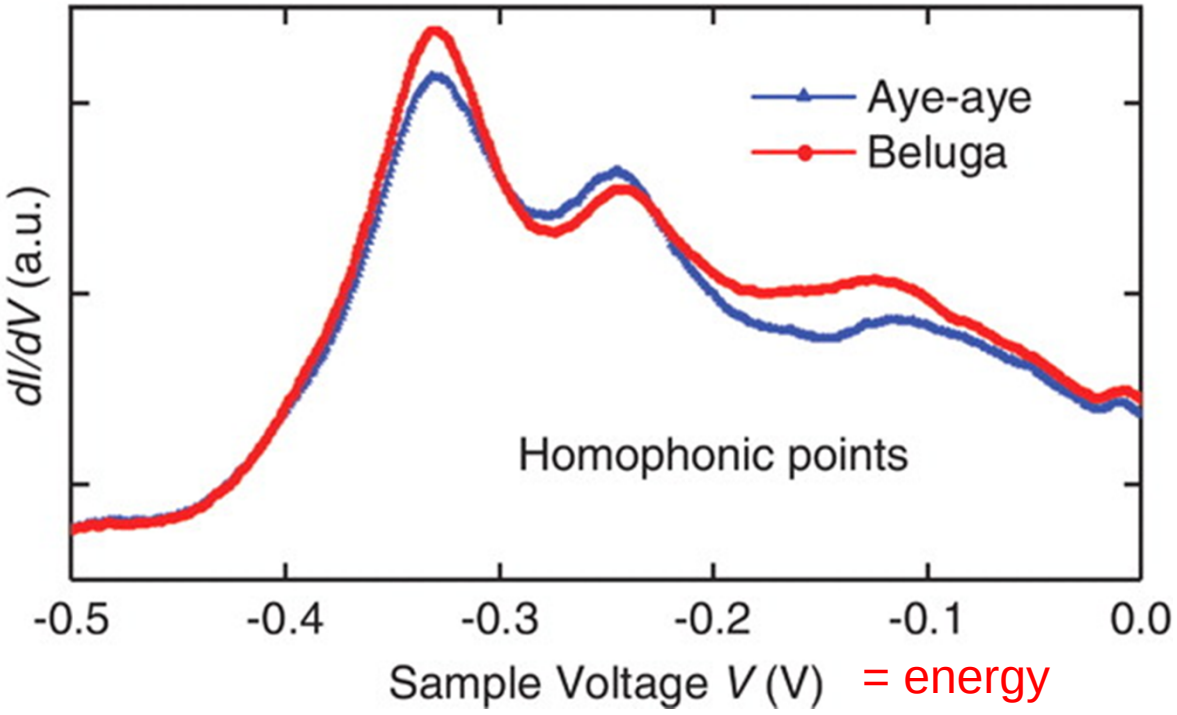

A
B
C
D
Bilby
Hawk
Broken Hawk
Sample Voltage V (V)
Eigenmode Number
ΔV (mV)
Isospectral Mode Frequency (THz)
-0.15
-0.20
-0.25
-0.30
-0.35
-0.40
-0.45
1 2 3 4 5 6 7 8
12
8
4
0
-4
65
58
51
45
40
34
20
15





boundaries formed by CO molecules on Cu surface

= density of electron states at that energy



= energy