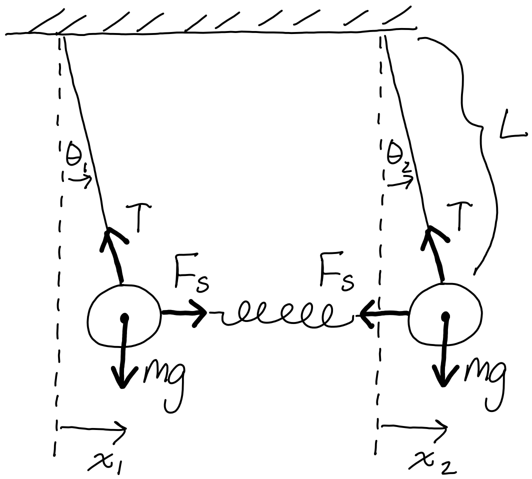
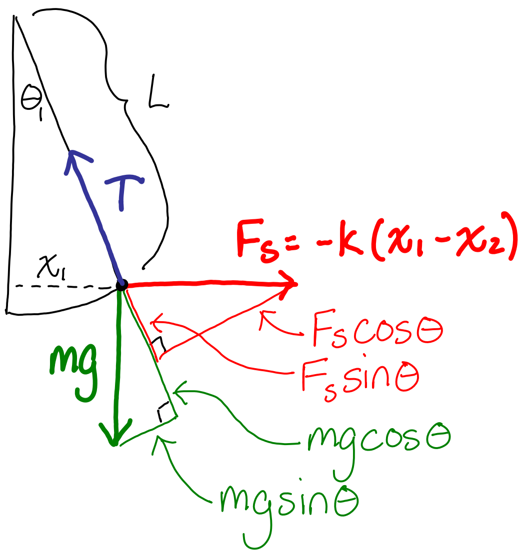


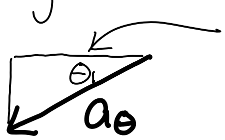

Consider the motion of the pendulum in polar coordinates. The arm of the pendulum has fixed length $L$, so there is no motion on the radial axis. So the radial forces must balance: $\sum F_r = T - F_s\sin\theta_1 - mg\cos\theta_1 = 0$

But on the angular axis, we have:

$$\sum F_\theta = F_s\cos\theta_1 - mg\sin\theta_1 = m\,a_\theta$$

Note that $\sin\theta_1 = x_1/L$ (exactly).

$a_\theta$ = tangential acceleration. We want to relate this to $d^2x_1/dt^2$




$$\frac{d^2x_1}{dt^2} = a_\theta\cos\theta_1 \Rightarrow a_\theta = \frac{1}{\cos\theta_1}\frac{d^2x_1}{dt^2}$$

So we have: $-k(x_1-x_2)\cos\theta_1 - mg\dfrac{x_1}{L} = \dfrac{1}{\cos\theta_1}\dfrac{d^2x_1}{dt^2}$

So far, this is exact (assuming that $F_s$ is exactly horizontal). Now we use the small angle approximation:

$$\cos\theta_1 = 1 - \tfrac{1}{2}\theta_1^2 + \ldots \approx 1$$

$$\frac{1}{\cos\theta_1} = 1 + \tfrac{1}{2}\theta_1^2 - \ldots \approx 1$$

And we arrive at the final $x$-axis eqn of motion:

$$\frac{d^2x_1}{dt^2} = -k(x_1-x_2) - \frac{mg}{L}x_1$$