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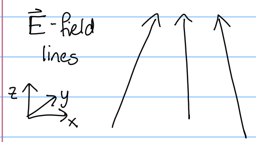
Physics 15b (Hoffman)
Lecture #23
Thurs, Dec 13, 2007

Title: "Magnetism in Matter"

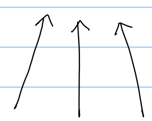
Summary of last lecture:

Electric fields & matter:

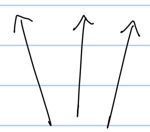
materials <u>always</u> polarize along the direction of $\vec{E}$
i.e. $\vec{P}$ is ALWAYS parallel to $\vec{E}$, even if its magnitude is non-linear



$\oplus\!\!\ominus$ $\vec{p} = p\hat{z}$
will move up, force toward stronger field



$\ominus\!\!\oplus$ $\vec{p} = -p\hat{z}$
will move down, force toward weaker field



$\oplus\!\!\ominus$ $\vec{p} = p\hat{z}$
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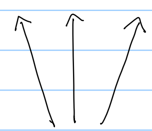

$\ominus\!\!\oplus$ $\vec{p} = -p\hat{z}$
will move up, force toward weaker field

This case will prevail for ALL real dielectrics, because $\vec{P}$ will always point in same direction as $\vec{E}$

Magnetic fields & matter:

We found empirically that some materials (e.g. bismuth) move towards weaker field, and some materials (e.g. $MnSO_4$) move towards stronger field.

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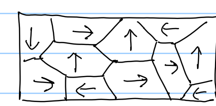
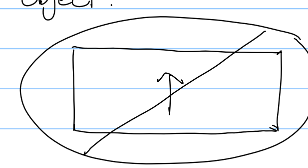
$\Rightarrow$ we <u>guess</u> that materials acquire a magnetization $\vec{M}$ in response to an applied $\vec{B}$, but that $\vec{M}$ is <u>not</u> always in the same direction as $\vec{B}$.

<u>Paramagnetic materials</u> (e.g. $MnSO_4$ and liquid O_2)
acquire $\vec{M}$ parallel to applied $\vec{B}$ (analogous to dielectrics)
$\rightarrow$ are attracted into regions of stronger $\vec{B}$-field

<u>Diamagnetic materials</u> (e.g. bismuth)
acquire $\vec{M}$ anti-parallel to applied $\vec{B}$ (no electrical analog)
$\rightarrow$ are repelled from regions of stronger $\vec{B}$-field

<u>Ferromagnetic materials</u> (e.g. iron)
already have a spontaneous $\vec{M}$, even in the absence of applied $\vec{B}$
(analogous to ferroelectrics $\leftarrow$ but these are rare)
$\rightarrow$ are attracted <u>or</u> repelled from regions of stronger $\vec{B}$-field, depending on initial orientation of material
$\rightarrow$ typically have their $\vec{M}$ aligned in local domains, not across the whole object:





Why? remember energy density $U = B^2/8\pi$ ...
$\rightarrow$ we can align the domains by applying a strong $\vec{B}$-field
$\rightarrow$ domains get easier to flip at higher T
$\rightarrow$ above Curie temperature T_c, domains lose their permanent magnetization altogether, and microscopic dipoles flop around like a paramagnet

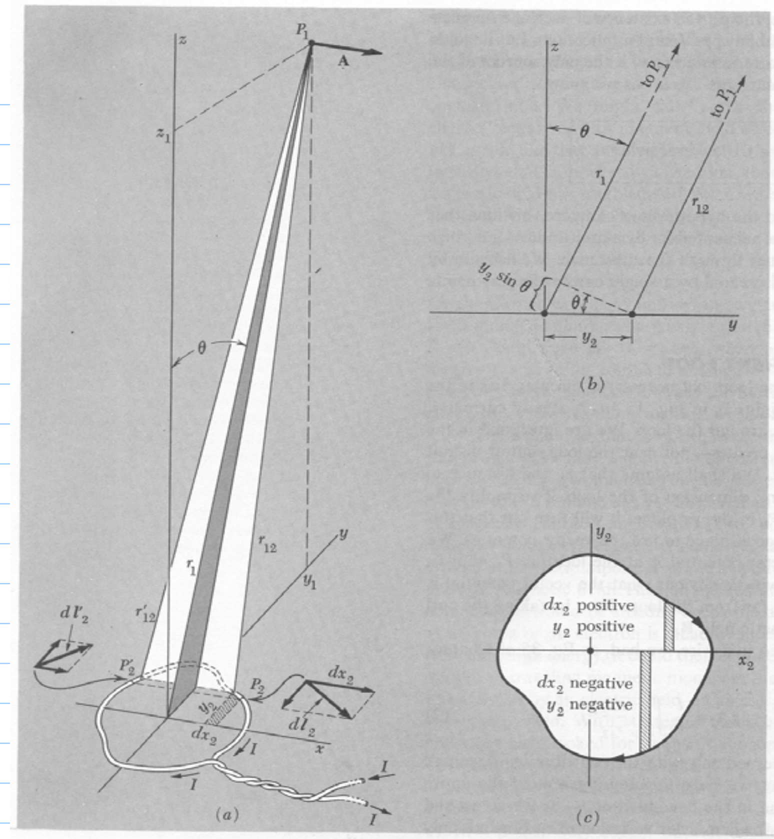
Barkhausen effect: as we bring a strong, uniformly aligned ferromagnet towards a multi-domain ferromagnetic wire, the domains in the wire start to flip to align with the strong $\vec{B}$ produced by the aligned ferromagnet. As each domain flips, it changes the surrounding $\vec{B}$-field slightly. The change in $\vec{B}$-field induces a current in the nearby coils via Lens' law. We attach amplifiers to the coils, then we can hear the domain flips!

Goals for today:

- magnetic dipole moment : $\vec{m} = I\vec{a}/c$
- force on dipole : $\vec{F} = (\vec{m}\cdot\vec{\nabla})\vec{B}$
- torque on dipole : $\vec{\tau} = \vec{m} \times \vec{B}$
- microscopic view of diamagnetism
- microscopic view of paramagnetism
- magnetic susceptibility χ_m
- magnetic field $\vec{H}$
- rewrite remaining 2 Maxwell eqns

Assert the calculation of magnetic dipole:

$\vec{m} = \frac{I\vec{a}}{c}$ for a current loop



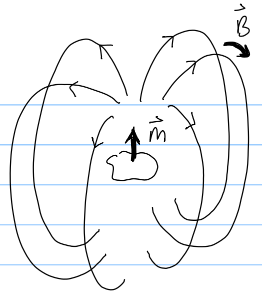
Compute $\vec{A}(P_1) = \frac{I}{c}\int_{\text{loop}} \frac{d\vec{l}_2}{r_{12}} = \frac{\vec{m}\times\hat{r}}{r^2}$

This calculation takes up pages 9–12 in lecture #22 notes; let's skip details and just assert the result for now.

where $\vec{m}$ = dipole moment $= \frac{I\vec{a}}{c}$

Does this make sense?

Yes, we expect that $\vec{B} = \vec{\nabla}\times\vec{A} = \vec{\nabla}\times\left(\frac{\vec{m}\times\hat{r}}{r^2}\right)$ should increase as I increases and as $\vec{a}$ increases (=more wire to produce $\vec{B}$).



$$\vec{B} = \vec{\nabla} \times \vec{A}$$

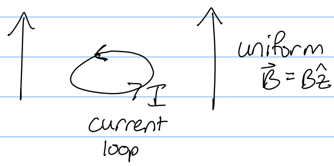
$$B_x = \frac{3mxz}{r^5}$$

$$B_y = \frac{3myz}{r^5}$$

$$B_z = \frac{m(3z^2 - r^2)}{r^5}$$

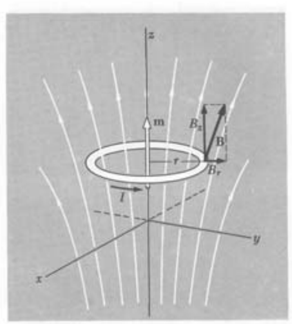
Force on a dipole

Uniform field:



$\vec{B}$ exerts a force outward on each increment of wire in the loop, but since it's a loop, each increment has a partner on the opposite side of the loop which is being pulled the opposite direction
$\Rightarrow$ net force is zero

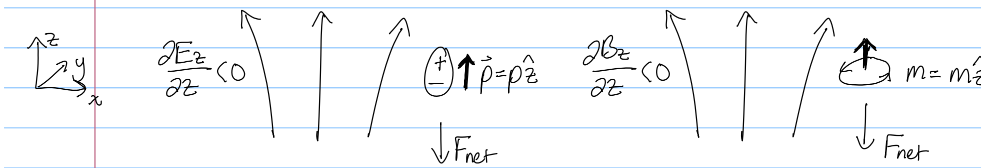
Non-uniform field:



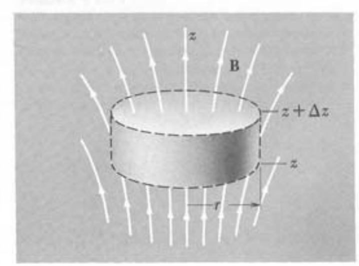
Because $\vec{\nabla} \cdot \vec{B} = 0$, field lines cannot stop and start. So if B_z is getting weaker as we go up in z, as shown, then the field lines must bend outwards in order to get farther apart. So a negative $\partial B_z / \partial z$ necessarily indicates an outward radial B_r.

At each current increment, the local B_r causes a downward force. So there will be a net downward force (towards stronger field) on the dipole.

Ah-hah! We see the parallel between $\vec{p}$ and $\vec{m}$:



Compute the magnitude of force on dipole:



$\vec{\nabla} \cdot \vec{B} = 0$, so total flux of $\vec{B}$ leaving any volume must be zero.

Consider this very small cylinder:

positive sign for flux leaving cylinder

$$\text{outward flux out sides} = +\underbrace{2\pi r \Delta z}_{\text{total area of sides}} \; B_r$$

radial field: assume it's constant over such a small area

$$\text{top \& bottom flux} = -\pi r^2 B_z(z) + \pi r^2 B_z(z + \Delta z)$$

negative sign for flux going in the bottom

plus sign for flux going out the top

$$= \pi r^2 \left[-B_z(z) + B_z(z) + \frac{\partial B_z}{\partial z} \Delta z \right] = \pi r^2 \frac{\partial B_z}{\partial z} \Delta z$$

$$\vec{\nabla}\cdot\vec{B}=0 \Rightarrow 2\pi r\,\Delta z\, B_r + \pi r^2 \frac{\partial B_z}{\partial z}\Delta z = 0$$

$$\Rightarrow B_r = -\frac{r}{2}\frac{\partial B_z}{\partial z}$$ (points outwards, because $\frac{\partial B_z}{\partial z}<0$ so 2 negatives cancel)

The force from B_r on an increment of loop $d\vec{l}$ carrying current I is: $d\vec{F} = -\frac{I\,dl\,B_r}{c}\hat{z}$

Integrating this over the loop gives us:

$$\vec{F} = -2\pi r\frac{I B_r}{c}\hat{z}$$

And plugging in B_r gives:

$$\vec{F} = -2\pi r\frac{I}{c}\left(-\frac{r}{2}\frac{\partial B_z}{\partial z}\right)\hat{z} = \underbrace{\frac{\pi r^2 I}{c}}_{m=\frac{Ia}{c}}\underbrace{\frac{\partial B_z}{\partial z}}_{\text{this is negative, so force is down}}\hat{z}$$

$$\Rightarrow \vec{F} = m\frac{\partial B_z}{\partial z}\hat{z}$$

The general case (which we won't prove) is: $\vec{F} = (\vec{m}\cdot\vec{\nabla})\vec{B}$

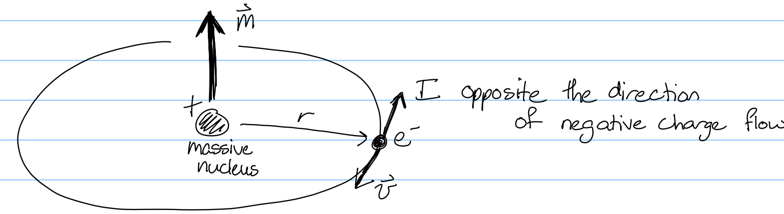
Torque on a dipole.

We proved it on a problem set several weeks ago:

$$\vec{\tau} = \vec{m}\times\vec{B}$$

Diamagnetism (microscopics)

We don't know quantum mechanics yet, so we use a simplified model of an atom which gives a qualitative feel for what's going on, and luckily gives us quantitatively the right answer.



Circulating electron makes a current loop

$$I = \frac{\text{total charge passing by fixed point}}{\text{unit time}}$$

$= e$ (# of times e goes around loop /unit time)

$= e$ (velocity/distance around loop) $= ev/2\pi r$

$$m = \frac{Ia}{c} = \frac{ev}{2\pi r}\cdot\pi r^2\cdot\frac{1}{c} = \frac{evr}{2c}$$

A side note: angular momentum of e^- is $L = m_e v r$

$$\Rightarrow \vec{m} = \underbrace{\frac{-e}{2cm_e}}_{\text{this is just made of fundamental constants} \equiv \text{"gyromagnetic ratio"}}\vec{L}$$ (note the opposite directions of $\vec{m}$ and $\vec{L}$)

$\vec{L}$ is a constant of motion (in the absence of external torques) so $\vec{m}$ will be conserved too, in the absence of external torques

In a macroscopic lump of material, all the atomic $\vec{m}$'s are pointed every which way, so there is no net $\vec{M}$ (ferromagnetic materials derive their net $\vec{M}$ from a different source).

But focus on atoms with e^- orbiting in xy plane:
What happens when we turn on $\vec{B} = -B_z\hat{z}$
Imagine that e^- is tethered to the nucleus at fixed r.
The force tethering the e^- must be:

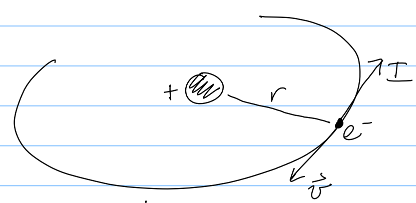
$$F_0 = \frac{m_e v_0^2}{r}$$

(from the mechanics of circular motion) where F_0 and v_0 are the initial force and velocity before turning on $\vec{B}$

While the $\vec{B}$-field is growing at $\frac{dB}{dt}$ in $-\hat{z}$ direction, Lenz' law applies:

$$\oint \vec{E}\cdot d\vec{l} = -\frac{1}{c}\frac{d\Phi}{dt} = -\frac{1}{c}\pi r^2 \underbrace{\frac{dB_z}{dt}}_{\text{this is negative}}$$

but lets drop signs and figure out directions from common sense



$$\Rightarrow 2\pi r E = \frac{\pi r^2}{c}\frac{dB_z}{dt}$$

$$E = \frac{r}{2c}\frac{dB_z}{dt}$$

$\vec{B}\downarrow$ as we increase B down, the current loop tries to provide a compensating B field up, so the current I must increase, which means that $\vec{E}$ points counter-clockwise and the velocity of the electron increases (which means F_0 must increase to hold it in).

The acceleration of the e^- is given by

$$m\frac{dv}{dt} = qE = \frac{er}{2c}\frac{dB_z}{dt}$$

$$m\int_{v_0}^{v_0+\Delta v} dv = \int_{B_0}^{B_0+\Delta B}\frac{er}{2c}\,dB \Rightarrow \Delta v = \frac{er}{2m_e c}\Delta B$$

The time dropped out, all that matters is total ΔB

$$\Delta m = \frac{er}{2c}\Delta v = \frac{e^2 r^2}{4m_e c^2}\Delta B$$

Sign check? As we increase $B\downarrow$, the velocity of the electron increases, which increases $m\uparrow$

$$\Rightarrow \Delta\vec{m} = -\frac{e^2 r^2}{4m_e c^2}\vec{B}$$

Ah-hah! We derived a diamagnetic "magnetizability", a microscopic reason why for some materials, $\vec{M} \propto -\vec{B}$. The exact constant of proportionality will depend on the details of the electron orbit (quantum mechanics) but this diamagnetic effect exists to some degree for <u>all</u> atoms.

If this were the only effect, we would expect <u>all</u> materials to be repelled by regions of stronger field.

So there must be another effect, a paramagnetic effect, which is stronger for some materials.

$$\vec{\tau} = \vec{m}\times\vec{B} = mB\sin\theta \Rightarrow \vec{M} = \frac{Nm^2}{k_B T}\vec{B}$$