Electric fields in matter

Q = charge = "monopole moment"

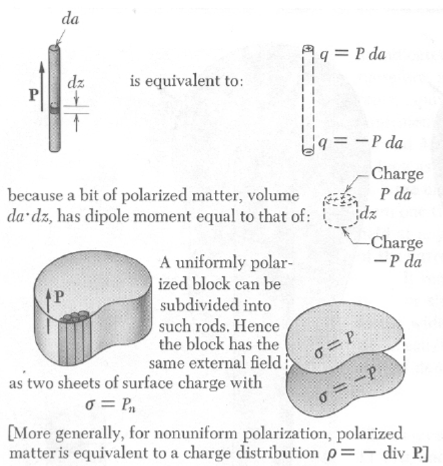
→ potential φ falls off like $1/r$
→ field $\vec{E}$ falls off like $1/r^2$

$\vec{p}$ = "dipole moment" = relative displacement of positive & negative charges

$$= \int_{\substack{\text{volume}\\ \text{of charge}\\ \text{distribution}}} \rho(\vec{r}\,')\,\vec{r}\,'\,dv = \sum_{\substack{\text{all point}\\ \text{charges } q_j}} q_j \vec{r}_j$$

→ potential φ falls off like $1/r^2$
→ field $\vec{E}$ falls off like $1/r^3$

$\vec{P}$ = "polarization" = dipole moment per unit volume
(e.g. $N\vec{p}$ where N = # of molecules/cm^3
and $\vec{p}$ = dipole moment of single molecule)



$\rho_{bound} = -\vec{\nabla}\cdot\vec{P}$

$\langle \vec{E} \rangle$ = average $\vec{E}$ inside material = exactly what you would calculate from this simple macroscopic charge distribution picture, ignoring the messy microscopics

Magnetic fields in matter

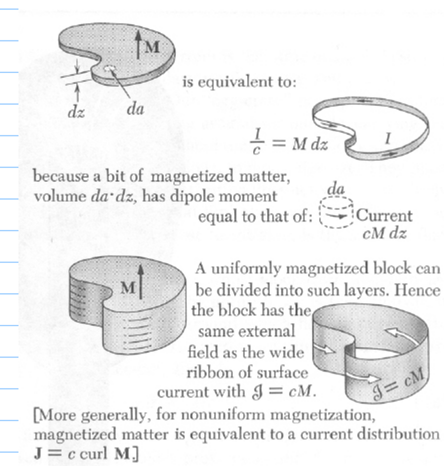
? = "magnetic monopole" - DOES NOT EXIST, as far as we know

$\vec{m}$ = "dipole moment" = size & strength of current loop

$$= \frac{Ia}{c} \;(\text{cgs}) \quad \text{or} \quad Ia \;(\text{SI})$$

→ field $\vec{B}$ falls off like $1/r^3$

$\vec{M}$ = "magnetization" = dipole moment per unit volume
(e.g. $N\vec{m}$ where N = # of molecules/cm^3
and $\vec{m}$ = dipole moment of single molecule)



$\vec{J}_{bound} = c\vec{\nabla}\times\vec{M}$

$\langle \vec{B} \rangle$ = average $\vec{B}$ inside material = exactly what you would calculate from this simple macroscopic current loop picture, ignoring the messy microscopics

α = "polarizability" = tendency of a non-polar molecule to acquire a dipole moment in an applied $\vec{E}$-field,
defined by $\vec{p} = \alpha \vec{E}$
typically $\alpha \sim a^3$ where a = dimension of molecule

χ_e = "dielectric susceptibility" = tendency of a macroscopic material to acquire a macroscopic polarization $\vec{P}$ in the presence of an applied $\vec{E}$
defined by $\vec{P} = \chi_e \vec{E}$
Note: not all materials obey this nice linear relation, but most do, for reasonable ranges of $\vec{E}$.
For a linear material: $\chi_e = \frac{\varepsilon - 1}{4\pi}$

2 kinds of materials:

non-polar: molecules have no spontaneous dipole moment, but applied $\vec{E}$-field moves negative charge cloud

- molecule always acquires $\vec{p}$ in the same direction as $\vec{E}$
- relatively weak effect
- exhibited by ALL molecules (molecules that are polar to begin with just have their dipole moment shifted slightly)

polar: molecules each have a dipole moment even when $\vec{E} = 0$, but they are all randomly oriented (cancelling) due to thermal agitation

- $\vec{E}$-field applies torque to each dipole
- dipoles tend to orient with $\vec{p}$ in the same direction as $\vec{E}$
- thermal agitation prevents all from completely aligning
  $\chi_e = \frac{Np^2}{k_B T}$
- relatively strong effect

Note: both types of materials acquire $\vec{P}$ parallel to $\vec{E}$

"magnetizability" - no letter exists to represent it

χ_m = "magnetic susceptibility" = tendency of a macroscopic material to acquire a macroscopic magnetization $\vec{M}$ in the presence of an applied field $\vec{H}$
defined by $\vec{M} = \chi_m \vec{H}$

2 kinds of materials:

diamagnetic: all electron spins are paired $\uparrow\downarrow$
no free spins
entire magnetic moment comes from "orbital" e^- motion
relatively weaker effect

orbital e^- motion changes in applied $\vec{B}$ due to Lenz' law
$\Delta\vec{m} = \frac{-e^2 r^2}{4 m_e c^2} \Delta\vec{B}$
note the negative sign!
$\Delta\vec{m}$ is opposite the direction of $\Delta\vec{B}$

paramagnetic: some unpaired, "free" electron spins
each spin carries its own spontaneous dipole moment $\hbar e/2m_e$, even when $\vec{B} = 0$
but they are all randomly oriented (cancelling) due to thermal agitation

- $\vec{B}$-field applies torque to each dipole
- dipoles tend to orient with $\vec{m}$ in the same direction as $\vec{B}$
- thermal agitation prevents all from completely aligning
  $\chi_{pm} = \frac{Nm^2}{k_B T}$
- relatively strong effect

Note: all materials acquire a small $\vec{M}$ anti-parallel to $\vec{B}$, from the diamagnetic effect
Some materials acquire an additional, stronger $\vec{M}$ parallel to $\vec{B}$ from the paramagnetic effect, which totall drowns out the weak diamagnetic effect

ε = "dielectric constant" - property of a material such that:
① if material is stuck between capacitor plates, the charge on the plates is increased from vacuum value Q_0 to $Q = \varepsilon Q_0$
② more generally, electric fields produced by free charges in the material are reduced by a factor ε with respect to vacuum
For a linear material, $\varepsilon = 4\pi\chi_e + 1$

$\vec{D}$ = "displacement current" $\equiv \vec{E} + 4\pi\vec{P}$
For a linear material, this reduces to
$$\vec{E} + 4\pi\left(\frac{\varepsilon - 1}{4\pi}\right)\vec{E} = \varepsilon\vec{E}$$
$\vec{D}$ is a convenient definition that allows us to simplify Maxwell's laws in matter.

You can sort of think of $\vec{D}$ as the "field produced by the free charges" but it's not a perfect analogy because $\vec{\nabla}\times\vec{D} \neq 0$

Note: $\vec{D}$ is not a very useful concept, because usually in the lab we do actually control $\vec{E}$ directly, because we can easily apply voltages with batteries or power supplies. If we control V, then $\vec{E}$ follows directly from $\vec{\nabla}V$, regardless of what material is there.

It's not actually so easy to control the free charge in the laboratory (only in our theoretical classroom minds).

Modified Maxwell's eqns:

general:

$$\vec{\nabla}\cdot\vec{D} = 4\pi\rho_{free}$$
$$\vec{\nabla}\times\vec{B} = \frac{1}{c}\frac{\partial\vec{D}}{\partial t} + \frac{4\pi}{c}\vec{J}_{free}$$

linear material:

$$\vec{\nabla}\cdot(\varepsilon\vec{E}) = 4\pi\rho_{free}$$
$$\vec{\nabla}\times\vec{B} = \frac{1}{c}\frac{\partial(\varepsilon\vec{E})}{\partial t} + \frac{4\pi}{c}\vec{J}_{free}$$

μ = "magnetic permeability" = property of a material such that the fundamental magnetic field $\vec{B}$ differs from the applied magnetic field $\vec{H}$ by $\vec{B} = \mu\vec{H}$. Also, $\mu = 1 + 4\pi\chi_m$
(This only makes sense for linear materials.)

$\vec{H}$ = "magnetic field H" $\equiv \vec{B} - 4\pi\vec{M}$
For a linear material, this reduces to
$$\vec{H} = \vec{B} - 4\pi\chi_m\vec{H} \Rightarrow \vec{B} = (1 + 4\pi\chi_m)\vec{H} = \mu\vec{H}$$
$\vec{H}$ is a convenient definition that allows us to simplify Maxwell's laws in matter

You can sort of think of $\vec{H}$ as the "field produced by the free currents" but it's not a perfect analogy because $\vec{\nabla}\cdot\vec{H} \neq 0$.

Note: $\vec{H}$ is a much more useful concept than $\vec{D}$, because usually in the lab we do actually control $\vec{H}$ directly, because we do supply and control the free currents.

It's this practical difference (the ease of controlling $\vec{E}$ directly but only controlling $\vec{B}$ indirectly via $\vec{H}$) that leads to the annoying asymmetry in the definitions:

$\vec{P} = \chi_e\vec{E}$	$\vec{M} = \chi_m\vec{H}$
$\vec{D} \equiv \vec{E} + 4\pi\vec{P}$	$\vec{H} \equiv \vec{B} - 4\pi\vec{M}$
$\varepsilon = 1 + 4\pi\chi_e$	$\mu = 1 + 4\pi\chi_m$
$\vec{D} = \varepsilon\vec{E}$	$\vec{B} = \mu\vec{H}$

Modified Maxwell's eqns:

general:

$$\vec{\nabla}\times\vec{H} = \frac{1}{c}\frac{\partial\vec{D}}{\partial t} + \frac{4\pi}{c}\vec{J}_{free}$$

linear material:

$$\vec{\nabla}\times\left(\frac{1}{\mu}\vec{B}\right) = \frac{1}{c}\frac{\partial(\varepsilon\vec{E})}{\partial t} + \frac{4\pi}{c}\vec{J}_{free}$$

Speed of light in matter:

$$\vec{\nabla}\cdot(\varepsilon\vec{E}) = 4\pi\rho_{free} \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\left(\frac{1}{\mu}\vec{B}\right) = \frac{1}{c}\frac{\partial(\varepsilon\vec{E})}{\partial t} + \frac{4\pi}{c}\vec{J}_{free} \qquad \vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

Assume this is a homogeneous material
→ no free charge or free currents

$$\vec{\nabla}\cdot(\varepsilon\vec{E}) = 0 \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\left(\frac{1}{\mu}\vec{B}\right) = \frac{1}{c}\frac{\partial(\varepsilon\vec{E})}{\partial t} \qquad \vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

Because it's homogeneous, we can take ε and μ out of derivatives:

$$\vec{\nabla}\cdot\vec{E} = 0 \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} = \frac{\varepsilon\mu}{c}\frac{\partial\vec{E}}{\partial t} \qquad \vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

Eliminate $\vec{B}$: $\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\frac{\varepsilon\mu}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$

$\Rightarrow$ speed of light in matter is $\frac{c}{\sqrt{\varepsilon\mu}}$

index of refraction $n = \sqrt{\varepsilon\mu}$